# Evaluating the relationship between urban road pattern and population using fractal geometry

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# Abstract

Roads pattern is closely related to the urban growth and population increase. However, few studies were conducted to evaluate the relationship between road pattern and population. In this paper, we evaluate the relationship between the road pattern and population density of San Antonio in Texas by using fractal geometry. Our approach is based on the box dimension of fractal theory for calculating the road dimension. The results show that the dimension of road is more volatile than that of population density. The regression analysis of the relationship between the dimensions of road and population density indicates that there exists a close relation between them.

## 1. Introduction

Urban is a system of high functional and visual complexity (Parish and Muller, 2001). Roads, as interactions between the elements of urban, give a very strong effect to the urban growth and population increase. Road networks alter the landscape spatial pattern (Forman and Alexander, 1998), for example, people tend to live along the road for traffic convenience. Thus, road distribution and its network structure are informative urban topics.

Another striking feature in the urban organization is population. Population research, especially population density research, provides a potentially strong, scientific framework for socio-economic analysis of urban form and spatial distribution (Mandelbrot 1983).

The relationship between urban road pattern and population were rarely investigated. Fractal geometry appears to be an appropriate method for this analysis. In fact, fractal geometry has been applied widely to urban pattern research. For instance, Batty and Longley (1994) calculated the fractal dimension of the boundaries and edges of cities as well as their pattern of land use and growth using the fractal geometry.

There are two reasons for choosing fractal geometry method for this work. First, the road and population of the satellite towns around a big city are similar in spatial distribution, which means they have self-similar attributes. Second, by using fractal analysis, we can acquire more quantitative spatial information of urban road and population to describe their relationship.

In this paper, we will evaluate the relationship between urban road pattern and urban population density. First, a new method, modified boxing fractal geometry, will be introduced to estimate the spatial pattern of urban roads quantitatively. We will then apply regression analysis to the relationship of dimensions of road and population density, which we consider of major importance in affecting the urban road system. This analysis will provide information about the fractal character of urban structure aiming at advancing related research on the urban road network and urban population more deeply.

## 2. Background

### 2.1 Fractals, Scaling and Fractal Dimension

Fractals are defined to be scale-invariant (self-similar or self-affine) geometric objects (Hastings and Sugihara, 1993). First fractal example was introduced by Mandelbrot (1967) in calculating the length of British Coastline. He argued that the coastline has a dimension somewhere between 1 (a straight line) and 2 (a plane), for example, 1.75. This is commonly referred to as fractal dimension (Figure 1).

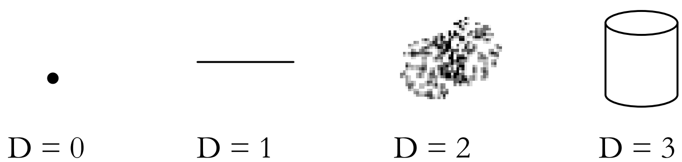


Figure 1. Increasing the dimension (from Hastings, and Sugihara, 1993)

Furthermore, the spatial pattern of the British coastline was examined across scales. He found individual sections of the coast have the same ‘patterns’ evident in the whole coastline, so that the major bays and headlands contain smaller bays and headlands. He refereed to this apparent ‘pattern’ replication across spatial scales as self-similar and called the object bearing self-similarity, “fractals” (Figure 2).

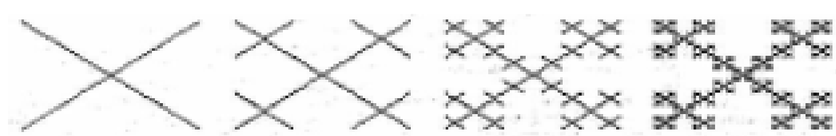


Figure 2. Self-similar across scales

### 2.2 Application of Fractal Geometry

Fractal geometry with scale parameterization offers significance for quantitative analysis of the spatial complexity of natural phenomena. For example, the fractal indices have been used extensively to estimate landscape complexity as a function of patch shape by computing the slope of a regression line between the natural logarithm of perimeter and area pairs calculated for all patches of interest (De Cola, 1989; Rex and Malanson 1990). However, this method requires enough perimeter and area pairs to accurately calculate the regression. Ricotta et al. (1998) calculated the constant of proportionality between the area and perimeter for the basic fractal equation to resolve the shortcoming of limited pairs.

Another interesting research about the relationship between sea-ice landscape and spatial patterns of polar bears was performed using fractal analysis (Frguson et al. 1998). In this article, the authors calculated the fractal dimension of polar bear movement pathways using the line segment method and the fractal dimension of ice cover for each block using the box counting method.

Olsen et al. (1993) applied the modified fractal dimension to measure the distribution of landscape diversity in a classified GIS image. This calculated dimension describes not only the diversity of patch shapes, but also patch juxtaposition and evenness by combining the number of landscape patches, their distribution, and shape.

2.3 Cities as Fractals: Simulating the spatial pattern

Coastline and man-made boundaries, of course, show self-similarity; furthermore the distribution of cities and their arrangement as central places, in terms of both size and spacing, illustrates distinct hierarchical ordering (Batty and Longley, 1991). Therefore, in dealing with the spatial structure, self-similarity should be considered. During the last decade, researchers have begun to examine the possibility of using fractal to capture the irregular shapes of developing cities (Peterson, 1996). Their efforts to study the urban pattern aim at uncovering the relationship between the physical form of a city to the behavior of its inhabitants and social-economic processes.

Batty and Longley (1994) noted that 'in defining the physical form delimiter of the city, its edge or boundary is the most obvious of its size and shape'. They calculated the fractal dimension of various towns and cities (notably Cardiff) by dividers method and compared the results with other researches. Researches reported in their book (*Fractal cities*) were conducted on fractals of urban boundaries and edges, land use, growth and form, and population density.

Longley et al. (1991) worked towards a consistent theory of urban growth and form in a system of urban settlements, combining allometric relationships and fractal geometries. In their article, they analyzed the development of analogies between the growth through the diffusion-limited aggregation (DLA) and the growth of urban areas, and offered some prospect for understanding how urban forms and densities evolve within a clearly specified pattern.

It is well known that the transportation system impacts the development and growth of cities. The closer to the city center, the higher density of the road is. Benguigui and Daoud (1991) analyzed the railway system by looking for the simple relation between the numbers of stations (N) as a function of the distance (R) to the center. They found that for the greater area railway system, N is roughly proportional to R1/2. They also mentioned that it would be very interesting to consider the population as a function of distance from a same point of view and to compare the fractal dimensions of the transportation system and of population density.

## 3. Study site and data preparation

### 3.1 Study site

My study area, the San Antonio and other parts of Bexar County around it, lies in the south-center of Texas. San Antonio is the third largest city in Texas; it covers about 412 square miles and has approximately 1,164,000 population (year 2000). The whole Bexar County has about 1,392,000 population (year 2000).

Figure 3 shows the seats of Bexar County and the major highway. San Antonio is the metropolitan city of Bexar County. As major centers of commerce and population, San Antonio forms the nuclei of traffic generation for the south-central section of Texas. The radial highway has an obvious hierarchical structure: when map scaled down, more detailed and similar road patterns appear. This means road pattern in this study area has the fractal character.

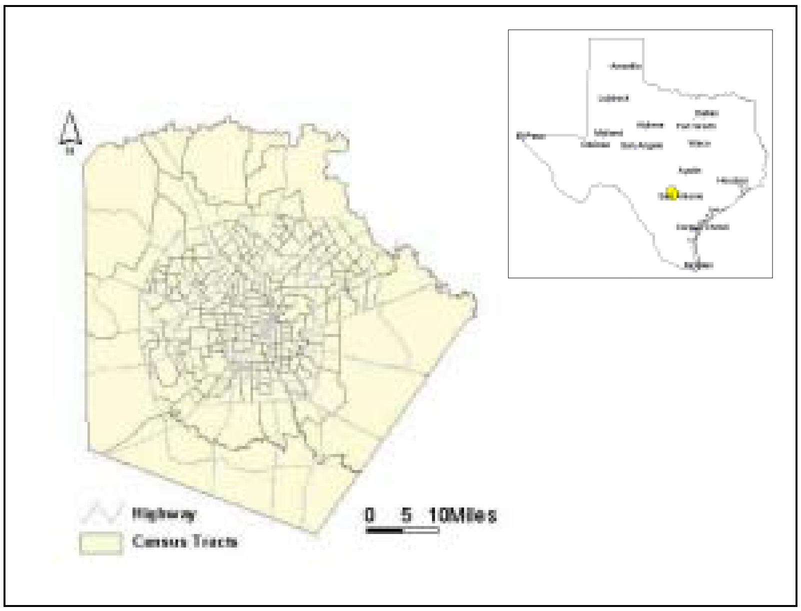



Figure3. The study area of Bexar County

### 3.2 Data Preparation

Urban road was acquired from the Geography network as shape file in 2000. The population density was acquired from the census data of 2000 in census tract unit. After transferring the road map from shape to coverage file in Arc/info, I clipped the road map into different circle areas by increasing the radius of buffer. In this research, we consider the population density as even inside one census tract unit.

## 4. Methods

Fractal method is one of the significant quantitative analysis methods of the spatial complexity of natural and social system, especially in the research of spatial organization of human activities across scales (Frankhauser, 1997). In fact, all fractal geographical entities exhibit a dimension relationship:

$$L^{1/1} \propto S^{1/2} \propto V^{1/3} \propto M^{1/D} \qquad (1)$$

Where L is the length of geographical entity, S is the area, V is the volume and M is any dimension, and D is the fractal geometry.

4.1 Road Pattern Fractal Analysis

The road network of the study area in the research area is analyzed using the modified boxing fractal geometry.

Figure 4 shows how the box dimension works. The box dimension of a subset X of the plane is defined similarly, by counting the numbers of small unit boxes, which intersect the subset X. At first, a small circle meets 4-unit square, if radius of the circle is doubled (b1), the circle will meet 16 unit squares, alternatively the cell size of unit squares in (a) is reduced by a linear factor of ½ (b2), it will meet 16 unit squares as well. We call this telescope-microscope principle.

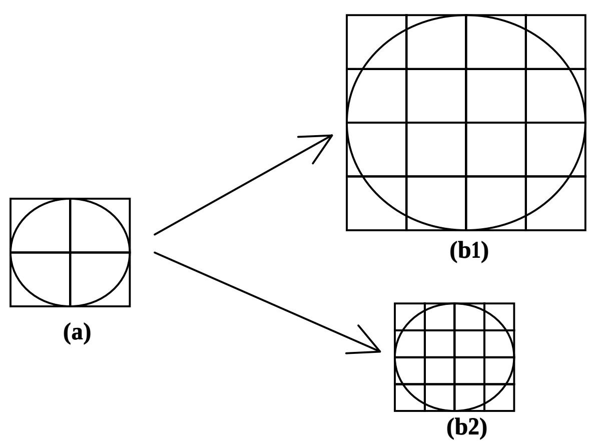



Figure4. The box dimension (from Hastings and Sugihara, 1993)

The similarity fractal is:

$$D = \lim_{\Delta s \to 0} \left[ -\log N(\Delta s) / \log \Delta s \right] \qquad (2)$$

Where the N (Δs) denotes the number of boxes which meet X in a grid of linear scale Δs.

In this paper, I use this model to calculate the fractal geometry of road by increasing the radius of circle and calculating the length of road in different radii. When the radius of circle R increases, the length of road L(R) increases correspondingly. Therefore the L(R)-R curve forms. We always use the change of ratio of this curve to indicate the fractal geometry.

The equation can be written as:

$$D(L_i) = \frac{\log[L(R_i) / L(R_{i-1})]}{\log[R_i / R_{i-1}]} \tag{3}$$

Where Ri is the radius at scale i, L(Ri) is the length of road at scale i, Ri-1 is the cell length of scale i-1, and L(Ri-1) is the length of road at scale of i-1.

4.2 Population Density Fractal Analysis

In the most basic form, population density can be considered as the scaling between population N(R) and area A(R); however the population N(R) can be defined as

$$N(R) = \gamma A^{\phi} = \gamma(\pi R^2)^{\phi} = \vartheta R^D \tag{4}$$

$$\rho(R) = \frac{N(R)}{A(R)} = \frac{\vartheta R^D}{\pi R^2} = \xi R^{D-2} \tag{5}$$

Where $\gamma$, $\vartheta$ and $\xi$ are constants of proportionality, D was 'effective dimension' (Takayasu, 1989), and in our research, D is the fractal dimension of population density (Batty and Longley, 1991). Then for any distance R from the center, D can be calculated from a log transform of this equation:

$$D(R) = 2 + \frac{\log \rho(R)}{\log R} \tag{6}$$

4.3 Correlation Analysis of Road Pattern and Population

Correlation and regression analyses are performed to investigate the relationship between the road pattern and population density. We will use log-likelihood ratio value tests to compare the natural logarithm of road and total population.

## 5. Results

The road dimensions are calculated by equation 3. First, we get the central point of Bexar County, and then buffer around the central point inside the map extent of Bexar County; we generate Buffers in fifty-one scales. The radius of circle begins from 1.5 kilometers and increases by 0.5 kilometers each time. By clipping the road by the buffers, we calculate the length of road in different buffer regions in Arc/info (figure 5). The population density in the buffer region is calculated based on the area and population density of census tracts, which locate in the buffer area. After calculating the total population in the buffer area, we get the population density of every buffer area through dividing the total population by buffer area.

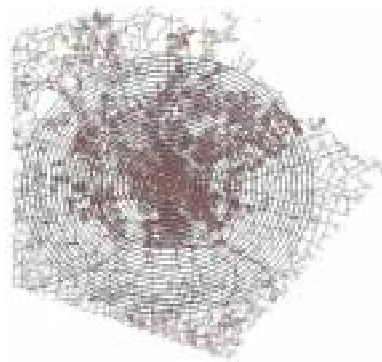


Figure5. The buffer on the road

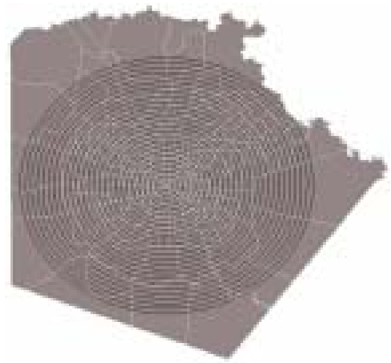


Figure6. The buffer on the population density

In general, as the scale progresses, the length of road and total population will increase as showed in Figure 5 and Figure 6, where the circles of buffer range from 1.5 kilometers to 26.5 kilometers by increasing 0.5 kilometer. If the road and population are distributed evenly in the whole research area, we could find that the length of road (LR) and total population (TP) are proportional to R. Furthermore, if their space distributions are compact, that is, if their distribution is constant, we would find that LR and TP are proportional to R2. However, for a fractal distribution, we can conclude that there exist relationships between the LR, TP and R, namely, LR and TP are proportional to $R^D$, where D is the fractal dimension.

Figure 7 shows the relationships between logarithm of total population and logarithm of radius of buffer. Two regimes are clearly divided by one turning point at the Log(R) ≈ 1.26: close to the center, there is a constant density region with the population increasing stably. When the distances are larger than approximately R ≈ 18.5 kilometers, the rate of increase in total population slows down. Interestingly, we find that the value of R at the turning point is close to the radius of the city of San Antonio. The result matches the real world very well. As the biggest city of Bexar County, San Antonio has almost 85% of the total population of the county. When the radius of buffer increases, it includes more area that is outside of San Antonio, and the population density decreases accordingly.

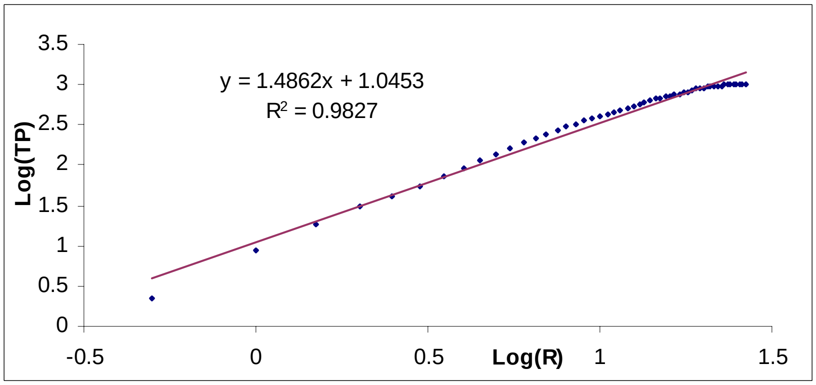



Figure7. Log-log of the total population as a function of radius of circle

Figure 8 is the logarithm of the length of road Log (LR) versus the logarithm of the radius of circles Log(R). Though the turning point is not as obvious as total population, there is still a slow-down in the increasing rate of length of road presented in this figure.

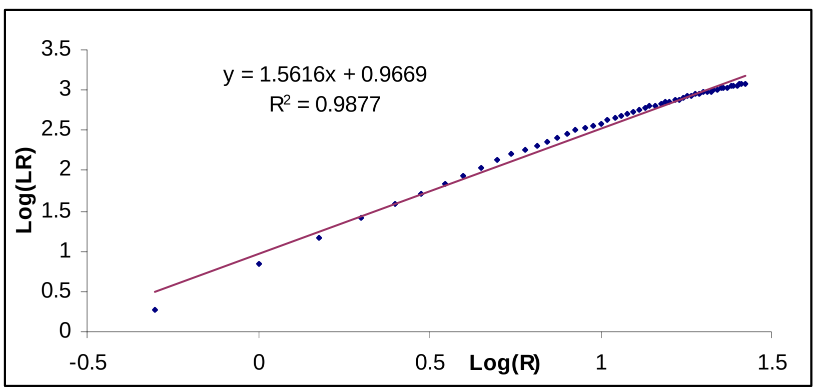



Figure8. Log-log of the length of road as a function of radius of circle

Comparing the figures 7 and 8, the result shows clearly the similar trend between the total population and length of road. This encouraging result motivates us to study more carefully on the relationship between the fractal geometry of road and population density to look for a quantitative explanation.

Table 1 presents the results of the fractal dimensions of road and population density in every scale, computed by equation (3) and equation (6) respectively.

In general, the bigger the radius of circle is, the smaller the fractal dimensions of road and population density are. This trend is especially clear in the dimension of population density, which decreases one by one slowly. We can see that the highest dimension of population density is 4.382 (when R = 1.5 kilometers), which means that people have filled in this center area greatly. Though the dimensions of population density fall within the range 4.5~1.7, we can see that most of values are around 2.0, thus suggesting that the 'true' dimension of population density should be close to 2.0.

The range of dimensions of road computed from equation (3) is much narrower than that of population density. Though some values are less than 1.0, they are still very near to 1.0. What is extremely interesting for this data set is that all the dimensions of road are less than 2.0, which suggests that roads do not fill in the entire two-dimensional space. With 38 out of the 51 values falling in the range of $0.8 < D < 1.5$, it is fairly convincing that the urban road has a 'true' dimension close to 1.0, with the likely value of 1.0~1.3.

Table1. Dimensions of road and population density

| | **1.5** | **2.0** | **2.5** | **3.0** | **3.5** | **4.0** | **4.5** | **5.0** | **5.5** | **6.0** | **6.5** | **7.0** | **7.5** | **8.0** | **8.5** | **9.0** | **9.5** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $D_{rd}$ | 1.905 | 1.951 | 1.893 | 1.740 | 1.832 | 1.833 | 1.672 | 1.833 | 1.756 | 1.686 | 1.615 | 1.533 | 1.592 | 1.498 | 1.457 | 1.364 | 1.280 |
| $D_{pd}$ | 4.382 | 3.262 | 2.831 | 2.618 | 2.501 | 2.430 | 2.383 | 2.342 | 2.309 | 2.283 | 2.260 | 2.236 | 2.211 | 2.188 | 2.168 | 2.148 | 2.127 |
| | | | | | | | | | | | | | | | | | |
| | **10.0** | **10.5** | **11.0** | **11.5** | **12.0** | **12.5** | **13.0** | **13.5** | **14.0** | **14.5** | **15.0** | **15.5** | **16.0** | **16.5** | **17.0** | **17.5** | **18.0** |
| $D_{rd}$ | 1.352 | 1.345 | 1.410 | 1.379 | 1.532 | 1.460 | 1.580 | 1.343 | 1.127 | 1.126 | 1.182 | 1.177 | 1.054 | 1.096 | 1.181 | 1.127 | 1.126 |
| $D_{pd}$ | 2.108 | 2.091 | 2.076 | 2.064 | 2.053 | 2.044 | 2.035 | 2.024 | 2.011 | 1.998 | 1.987 | 1.976 | 1.964 | 1.954 | 1.944 | 1.935 | 1.927 |
| | | | | | | | | | | | | | | | | | |
| | **18.5** | **19.0** | **19.5** | **20.0** | **20.5** | **21.0** | **21.5** | **22.0** | **22.5** | **23.0** | **23.5** | **24.0** | **24.5** | **25.0** | **25.5** | **26.0** | **26.5** |
| $D_{rd}$ | 1.222 | 1.249 | 1.008 | 0.893 | 1.011 | 0.941 | 0.898 | 0.861 | 0.917 | 1.053 | 1.055 | 0.967 | 0.863 | 0.791 | 0.901 | 0.851 | 0.893 |
| $D_{pd}$ | 1.920 | 1.912 | 1.904 | 1.894 | 1.884 | 1.874 | 1.863 | 1.853 | 1.842 | 1.832 | 1.822 | 1.813 | 1.803 | 1.793 | 1.784 | 1.775 | 1.766 |

Note: $D_{rd}$ is the fractal dimension of road, $D_{pd}$ is the fractal dimension of population density. We use the radius of circle to indicate the scales, radius unit: kilometer.

Though the general trends of the fractal dimension of road and population density of the study area are similar, there is still difference between them. From Figure 9, we can see that the decrease of the dimensions of road is more volatile than that of population density. Unlike the curve of dimensions of population density, which is very smooth, the curve of dimensions of road show some rebounds through the general declining process. In fact, if a further detailed analysis is conducted, the values of the dimension around the edge of the city may show more instability, since city and its road system are likely to be developing most rapidly at the edge of cluster.

Comparing to the curve of road dimensions, the curve of population density decreases sharply in the vicinity of the original point. It indicates that the central area of buffer has most dense population. Because the population density is calculated based on the census tract unit and the population density is supposed to be even inside one census tract, the difference between the origin and the edge of city has been averaged.

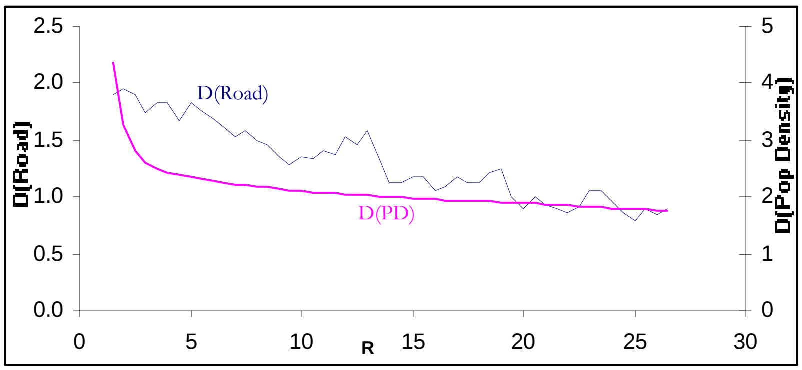
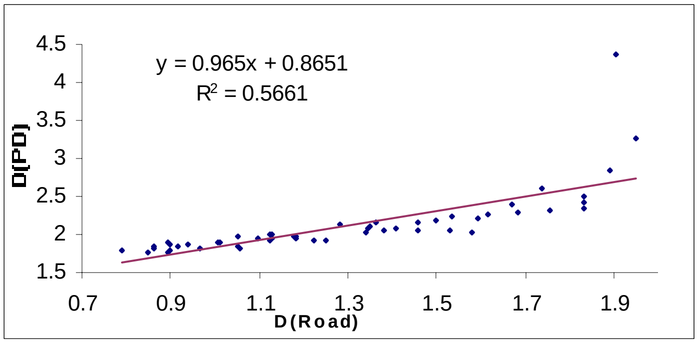



Figure9. A comparison of Dimensions of Road and Population Density

The regression analysis for the relationship between road dimension and population density is shown in Figure 10. The regression R2 statistics in Figure 10a is calculated on all the scales and Figure 10b is calculated after removing the two smallest buffers. This regression analysis generally confirms our expectations. Although the coefficient of determination in Figure 10a is only 0.5661, after removing the first two numbers, we can see that coefficient of determination improves to 0.8381, indicating that the close correlation between road and population density exists in this research area.

a




b

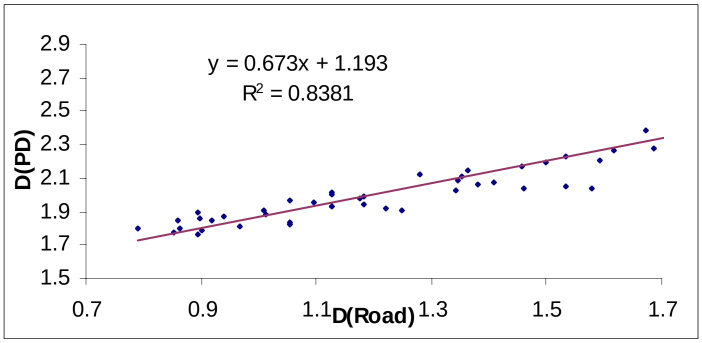



Figure10a. The relation between the dimensions of Road and population density
Figure10b. The relation between the dimensions of Road and population density (removing the two smallest buffers)

## 6. Conclusion

Research on the relationship between urban road and population has been rare for lacking the appropriate method. Fractal geometry is a promising technique to support research in this field. In this study, we explore the feasibility of fractal geometry in describing the relationship between urban structures and population factors.

Fractal geometry provides a practical advantage for urban study. For example, we have used the allometric growth for urban growth research before. However, the allometric growth implies constant urban densities over time and space, while the fractal growth implies a deterministic or stochastic growth.

In this paper, we calculate the road dimensions based on the box dimension by increasing the radius of buffer to cover more roads in the research area. In fact, this is one of box dimension methods; another way of halfing the cell size is to improve the resolution of background. We could try different methods to calculate the road dimension. Such research might enable the underlying theoretical assumptions of fractal model to be made more realistic and to incorporate more methods for our road pattern research.

In urban area, there are different types of roads. Major roads have higher impact on urban population size. For example, highway is more effective than village road in its effect to increase urban population. However, in our research, we treat them as the same level instead of classifying them first. Therefore, it might have impacted the overall result to a degree.

From the fractal dimension research of road and population density, we find perhaps the most important value of this analysis is not in demonstrating the fractal results produced by both theoretical and empirical analysis, but in the way of collecting data and defining dimension. Although these results are encouraging and confirm our initial hypotheses, there are still a lot of improvement can be done. We take the population density as constant in every census tract. In fact, the distance from a central seed site attenuates the population density and density decreases consequently. Strictly, we need more detailed data based on smaller spatial unit. Another deficiency is that we only calculate the dimensions located in the buffer area. Because of the irregular shape of county, there is still some space of the study area place not included in our research (see Figures 5 and 6). So if we want to know the dimension around the edge of county, we need to collect more data around Bexar County.

Finally, it is important to note that the self-similar structure of urban road pattern implies that a common scheme is used for the growth and form of urban area. When city grows, the fractal dimensions seldom keep constant. From this point of view, it would be extremely interesting to compare older data with present data, and check how the fractal dimensions change with time. This may be useful in predicting the urban pattern and population in the future. Similarly, it will be very interesting to see how the population density and its fractal dimension change with time.

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