Spatializing Zipf's Law in the Dynamic Context: US Cities 1960-2000[1]

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Abstract
This research investigates the U.S.'s urban system evolution through spatializing Zipf's law in the dynamic context. First, the paper revisits urban system dynamic theories and Zipf's law. Then several methods of exploratory space-time mapping are illustrated from regional, rank group, individual and driving forces perspectives, which show some interesting patterns otherwise ignored by traditional approaches which focus on the overall pattern. The paper concludes with evaluating applicability of Zipf's law in the space-time perspective and future research directions.

Keywords: Zipf's Law, Space-Time, US Cities 1960-2000

1. Urban System Evolution and Zipf's law

Dramatic urbanization has profoundly altered the surface of the earth at scales ranging from the local to the global (Turner II et al. 1990, Imre and Bogaert 2004). Many issues such as terrorism, public health, and globalization intensify the concerns how cities grow in population size and how they are connected to each other. There is a vast and growing literature exploring the nature of city size distribution at different geographic scales (Beckmann 1958; Carroll 1982; Parr 1985; Cheshire 1999; Chen and Zhou 2003; Batty 2005). As a complex system, the city changes through a process of positive feedback in which their size is a function of their growth rate at any point in time (Batty 2005). In 1941, Harvard linguistics professor George Kingsley Zipf finds that the frequency of the *k*th most common word in a text is almost proportional to $1/k$. Zipf further illustrates this discovery as a rank-size distribution wherein any city's population increments are divided by its rank on a log-log plot (Fig 2 in the following section). Ranks and sizes are closely related, resulting in near linear patterns on log-log plots for many socio-economic phenomena (Zipf 1949).

Gabaix (1999) notes that "Zipf's law for cities is one of the most conspicuous empirical facts in economics, or in the social sciences generally." Zipf's law has been gaining continuous interests for its accurate description of city size distributions in many different countries and at different times within a country (Rosen and Resnick, 1980; Mills and Hamilton, 1994; Fujita *et al.*, 1999; Batty 2003; Xie and Ye 2005). There are many renewed interests and efforts of applying Zipf's Law to various topics in the last half-century with publications from entertainment newspaper to the journal of Science, such as languages (Ferrer-i-Cancho 2006), web access statistics (Huberman et al. 1998), statistical physics (Blank and Solomon 2001), firm size (Fujiwara *et al.*, 2004), among others.

Zipf's law for city size distribution can be written as

$$\ln y = \ln a - b \ln r \qquad (1)$$

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Where *y* is the city size of rank r; *r* is the city size by rank, with *r*=1 meaning the largest city. Constant *a* equals the size of the largest city and *b* is the known as the Zip's component. When *b* equals 1, Zipf's law becomes the rank size rule, that is, that the second-largest city is one-half the size of the largest one, and so on. A smaller Zip's component indicates a more even distribution of city sizes.

Yet, this law has received much criticism due to its aspatial nature and the lack of consistent and well-defined explanations of the processes that govern city size distributions (Casetti 1972; Malecki 1975; Carroll 1982; Fan 1999). Sheppard (1982) questions whether other distributional functions might fit closer to the data. Harris (2004) suggests a research agenda on the structure of systems of cities for a scientific understanding of "causal interconnections that will support an effort to predict the future of cities, and to evaluate the impact of proposed methods for guiding that future". Hence, a new framework is suggested to investigate the space-time dynamic of urban system using exploratory geocomputational methods and visualization techniques.

2. Objectives and Approaches

This paper introduces new advances in spatial econometrics methods for the exploratory analysis of rank-size distribution dynamic (Rey 2004). The decadal urban size data in the U.S. are panel data between 1960 and 2000 for the same 160 cities from UN World Urban Prospects. First, the points that represent cities are converted to Voronoi polygons to build a spatial matrix. The polygon map will be used to represent the spatial distribution of cities with the color indicating the size of the urban population whose city dominates the polygon. More large value outliers are identified in the Year 2000 map (the right view) in the Figure 1.

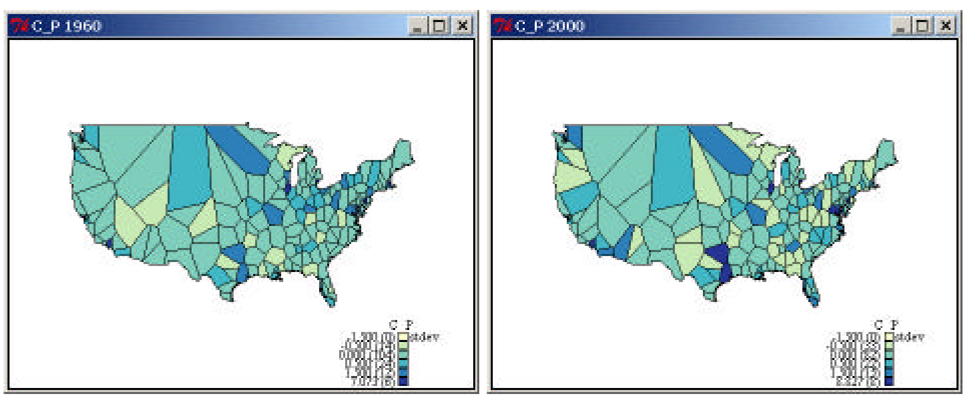


Figure 1 Voronoi Polygons of Urban Size (Classified by Standard Deviation), 1960 and 2000

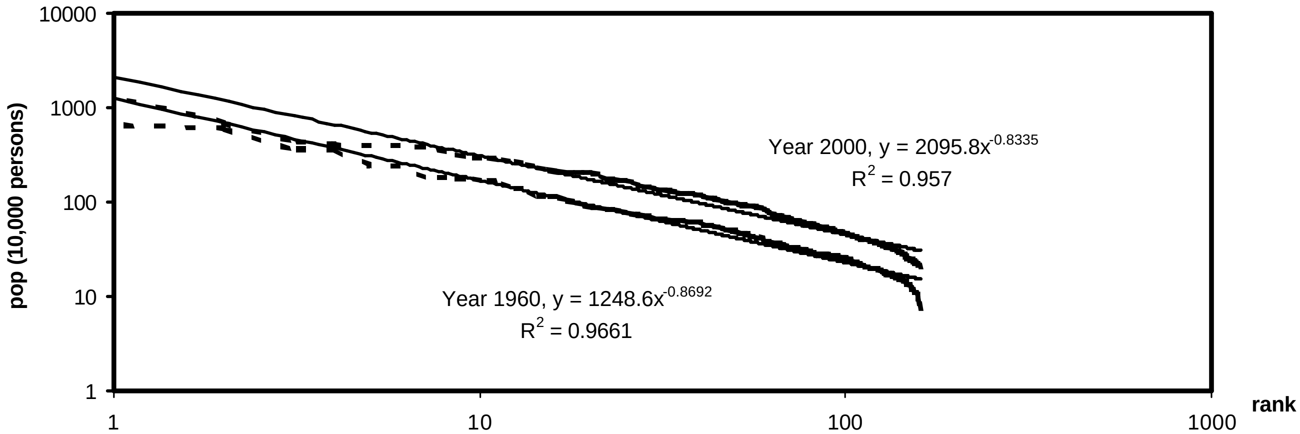


Figure 2 Zipf's Law Plot
Notes: The dashed line shows the data points, and the solid line is the fitted power line from Zipf's law (Y axis is the city size and X axis is the city size by rank; use Year 1960 and Year 2000 as example).

A log-log plot based on the Zipf's Law equation is drawn (Figure 2). Figure 2 demonstrates the high R square for both of the urban size distributions at the two time points (starting year 1960 and ending year 2000; Year 1970, 1980 and 1990 illustrate very similar general trends so they are not included in this figure). The two Zipf's components are around 0.8~0.9 at a very significant level, close to rank size rule. The upper ranks and bottom ranks are below the expected trend lines while the middle ranks are on the other side. However, these differences are smoothed into the general trend. Also, the two trend lines are almost parallel, which hide the rich details of spatial dynamics taking place in the four decades when dramatic urbanization has been witnessed.

Hence, we suggest some space-time mapping methods to provide insightful extensions to existing aspatial static approaches using STARS (STARS: Space–Time Analysis of Regional Systems): a NSF-supported open source software developed by Rey and his research group at San Diego State university (Rey and Janikas 2006). Interactive visualization is supported by dynamically linking spatial and statistical graphics.

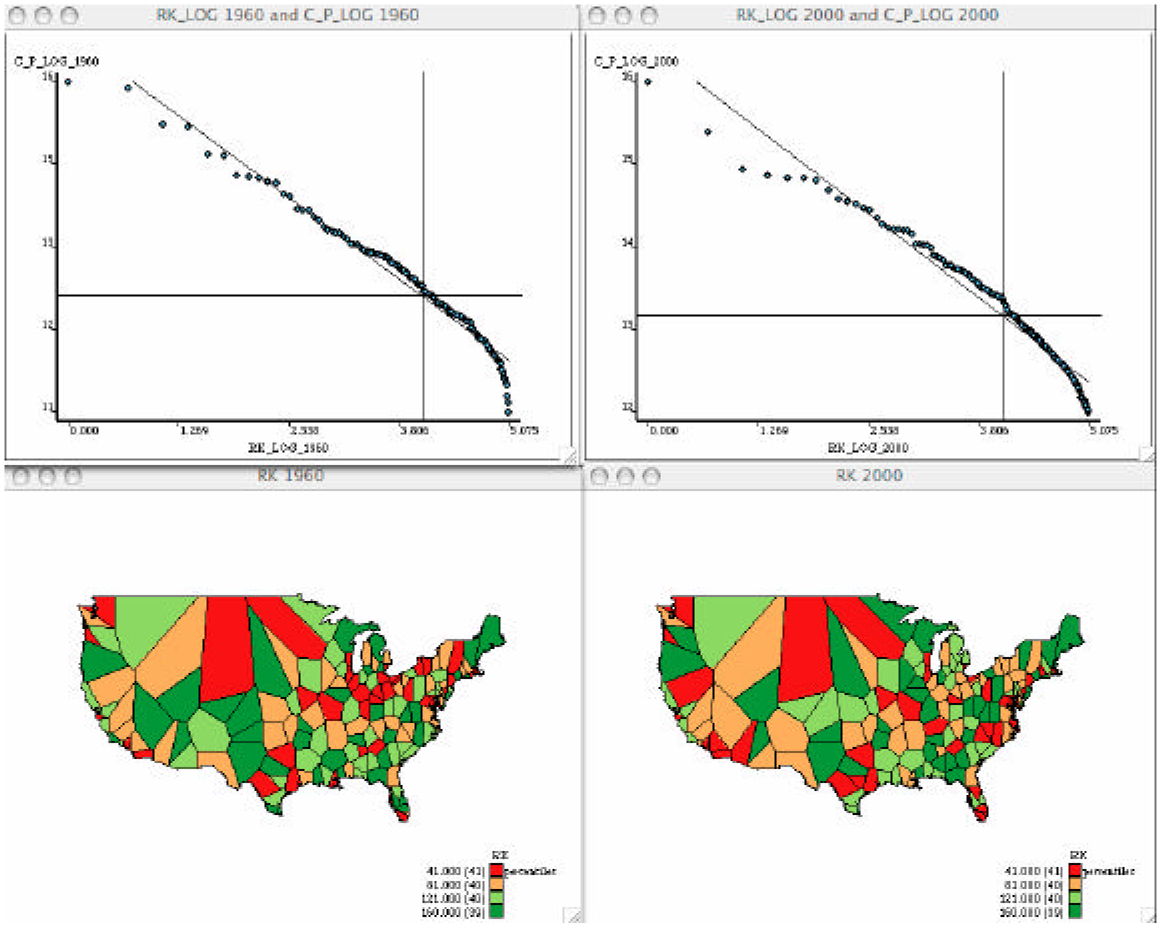


Figure 3

Figure 3 summarizes the above discussions by plotting the contents of Figure 1 and 2 linking the scatter plot and rank diverging mapping. The upper two maps in Figure 3 are the rank and size in a log-log scatter plot (1960 and 2000). The bottom two are diverging mappings of urban size (from red to green, urban size decreases). With selecting of any city/cities in one of the views, the corresponding city/cities will be highlighted. The bottom view shows a dramatic change of the rank distribution over the forty years while the upper view is almost identical between the two time points. Obviously, the static Zipf's components cannot explain the urban system dynamic between the two years.

Figure 4 summarizes the evolution of rank-size distributions at 10-year intervals beginning in 1960 using global T and Moran's I. Urban population is more evenly distributed among the cities, as indicated by the decreasing global T. Global Moran's I is moving from negative to positive spatial autocorrelation (the expect value is –0.006), which means that adjacent areal units have more similar values or characteristics. Though the Moran's I is still very small (the spatial dependence of size distribution slightly change), it might be a change of the way how cities "talk" to each other (from negative to positive). The following four perspectives will reveal the changes behind the scene.

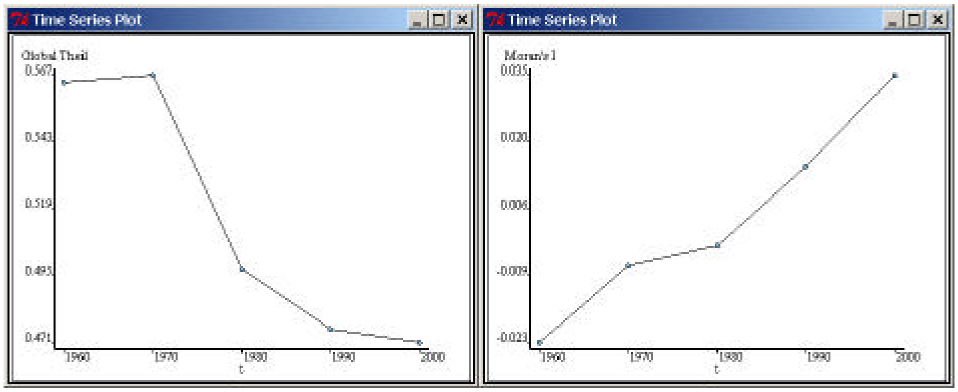


Figure 4 Global Theil and Global Moran's I

2.1. Regional Spatial Dynamic

The illustration of emergence and decline of cities within the aggregate system challenge the static and fixed-scale interpretation of conventional urban system theories. Thus, to explore urban system in sub-regions sharing similar social economic features with time-series data in a country might shed more lights on the system evolution (Ye and Xie 2006). In this subsection, two regional perspectives are examined, including local regime and census/BEA regions. Figure 5 shows the local regimes based on the LISA scatter plots in 1960 and 2000.

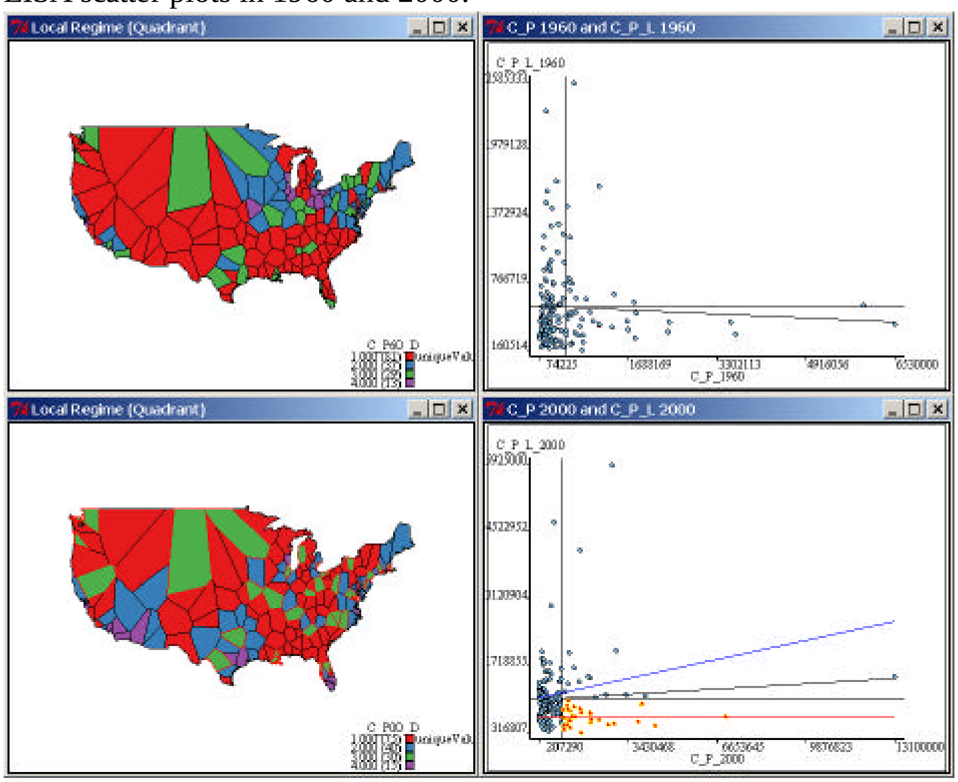


Figure 5. Local Regime and LISA Scatter Plot of Urban Size in 1960 (upper view) and 2000 (lower view)

Note: Four colors are used on the left maps to indicate four types of urban size value clusters (Red: Low-Low, Magenta: High-High, Green: High-Low, Blue: Low-High). On *the bottom view, the cities (points) in the High-Low part is selected which makes the corresponding polygons on the left map highlighted.*

Three geographic division systems are used to decompose the urban size distribution (Table 1, Table 2, Table 3 and the right view of Figure 6).

REGION	STATES
Northeast	Connecticut, Maine, Massachusetts, New Hampshire, New Jersey, New York, Pennsylvania, Rhode Island, Vermont
Midwest	Illinois, Indiana, Iowa, Kansas, Michigan, Minnesota, Missouri, Nebraska, North Dakota, Ohio, South Dakota, Wisconsin
South	Alabama, Arkansas, Delaware, Florida, Georgia, Kentucky, Louisiana, Maryland, Mississippi, North Carolina, Oklahoma, South Carolina, Tennessee, Texas, Virginia, West Virginia
West	Arizona, California, Colorado, Idaho, Montana, Nevada, New Mexico, Oregon, Utah, Washington, Wyoming

Table 1 Census Regions in Four

DIVISION	STATES
Northeast	Connecticut, Maine, Massachusetts, New Hampshire, Rhode Island, Vermont
Middle Atlantic	New Jersey, New York, Pennsylvania
East North Central	Illinois, Indiana, Michigan, Ohio, Wisconsin
West North Central	Iowa, Kansas, Minnesota, Missouri, Nebraska, North Dakota, South Dakota
South Atlantic	Delaware, Florida, Georgia, Maryland, North Carolina, South Carolina, Virginia, West Virginia
East South Central	Alabama, Kentucky, Mississippi, Tennessee
West South Central	Arkansas, Louisiana, Oklahoma, Texas
Mountain	Arizona, Colorado, Idaho, Montana, Nevada, New Mexico, Utah, Wyoming
Pacific	California, Oregon, Washington

Table 2 Census Regions in Nine

REGION	STATES
Northeast	Connecticut, Maine, Massachusetts, New Hampshire, Rhode Island, Vermont
Mideast	Delaware, Maryland, New Jersey, New York, Pennsylvania
Great Lakes	Illinois, Indiana, Michigan, Ohio, Wisconsin
Plains	Iowa, Kansas, Minnesota, Missouri, Nebraska, North Dakota, South Dakota
Southeast	Alabama, Arkansas, Florida, Georgia, Kentucky, Louisiana, Mississippi, North Carolina, South Carolina, Tennessee, Virginia, West Virginia
Southwest	Arizona, New Mexico, Oklahoma, Texas
Rocky Mountains	Colorado, Idaho, Montana, Utah, Wyoming
Far West	California, Nevada, Oregon, Washington

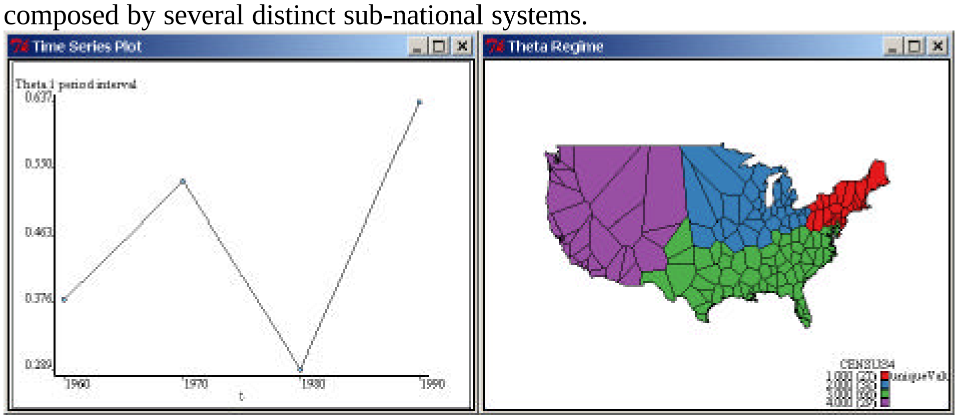
Table 3 BEA Regions in Eight

Rey (2004) suggests a measure to decompose distributional mobility to analyze the amount of spatial clustering. Let $\theta_{i,t}$ represent the rank of city i's urban size in year t. Then between any two periods, a scalar measure of spatial clustering of distributional transitions is:

$$\Theta_{t1-t0} = \frac{\sum_R \left|\sum_{i \in R} \theta_{i,t1} - \theta_{i,t0}\right|}{\sum_i \left|\theta_{i,t1} - \theta_{i,t0}\right|}$$

where R is one of a set of exhaustive and mutually exclusive groups of states (Census4, Census9 and BEA8 on Table 1,2,3 will be used as three scenarios). The denominator of this measure is the sum of the absolute rank changes over the period. Theta should be between 0 and 1. If movements in the urban size distribution are cohesive within regions, this measure should be closer to 1, which means the rank change is competitive across the regions. With less cohesion, the measure approaches 0.

Figure 6 applies the regional cohesion measure to the decadal rank changes in urban size. The value of Theta (left views) has the general trend of becoming larger, which means that the rank movement direction in one region becomes more similar (moving up or moving down at the same time period), that is, geography matters in the growth. In all of the three division systems, the Theta in 1990-2000 (the value above 1990 in the X axis) is the largest one, which suggests 1990s is the time period when the rank movement is most homogenous at the sub-national level. Hence, it justifies the assumption that the overall national urban system is composed by several distinct sub-national systems.



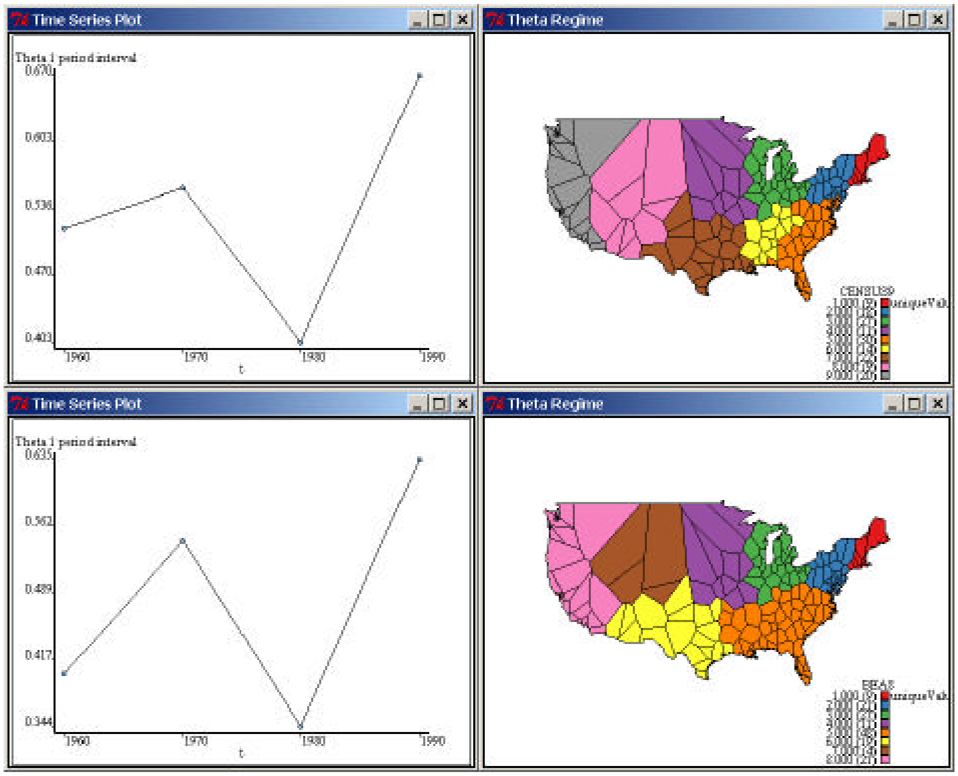


Figure 6. Spatial Cohesion Index: 10-year interval (Left) and corresponding Region Division (Right)
Note: Census Four regions, Census Nine Regions and BEA Eight Regions (from top to bottom)

2.2. Rank Group Spatial Dynamic

In this section, the overall spatial dynamic is decomposed into rank groups. Figure 7, 8 and 9 show the performance of ranks at the three sections of ranges from 1 to 160 (please notice the X axis uses log), that is, the 160 cities are divided into three groups with same amounts of cities based on their ranks in 1960. Through tracing their statuses in 1970, 1990 and 2000, we identify the three rank groups' dynamics from a forward perspective (1960-2000). The similar approach is applied when we examine the rank groups' dynamic from a backward perspective (2000-1960) (Figure 10, 11 and 12). The middle rank group shows the largest variation at both of the two perspectives, and these figures visualize the extent they move across the rank range.

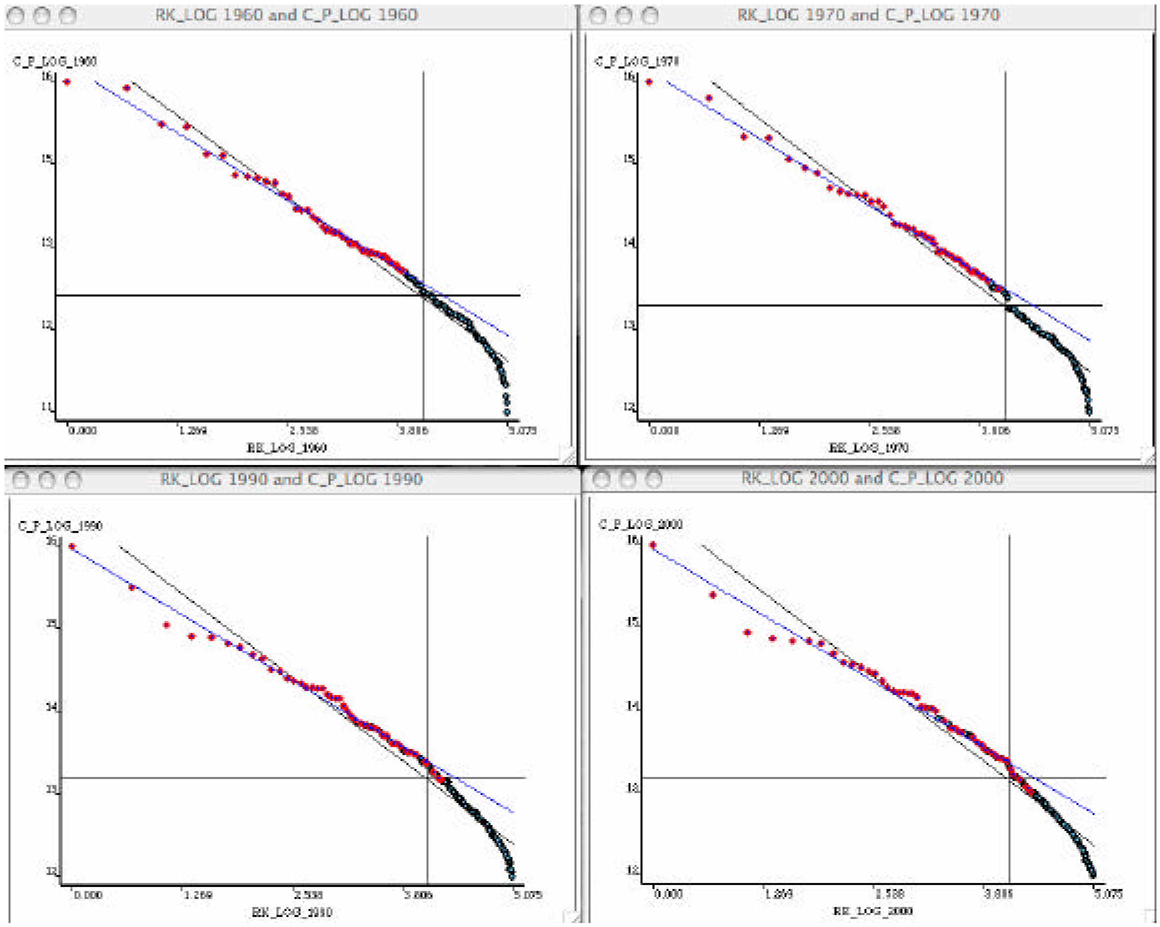


Figure 7. Upper Rank Dynamics from a Forward Perspective (1960-2000)

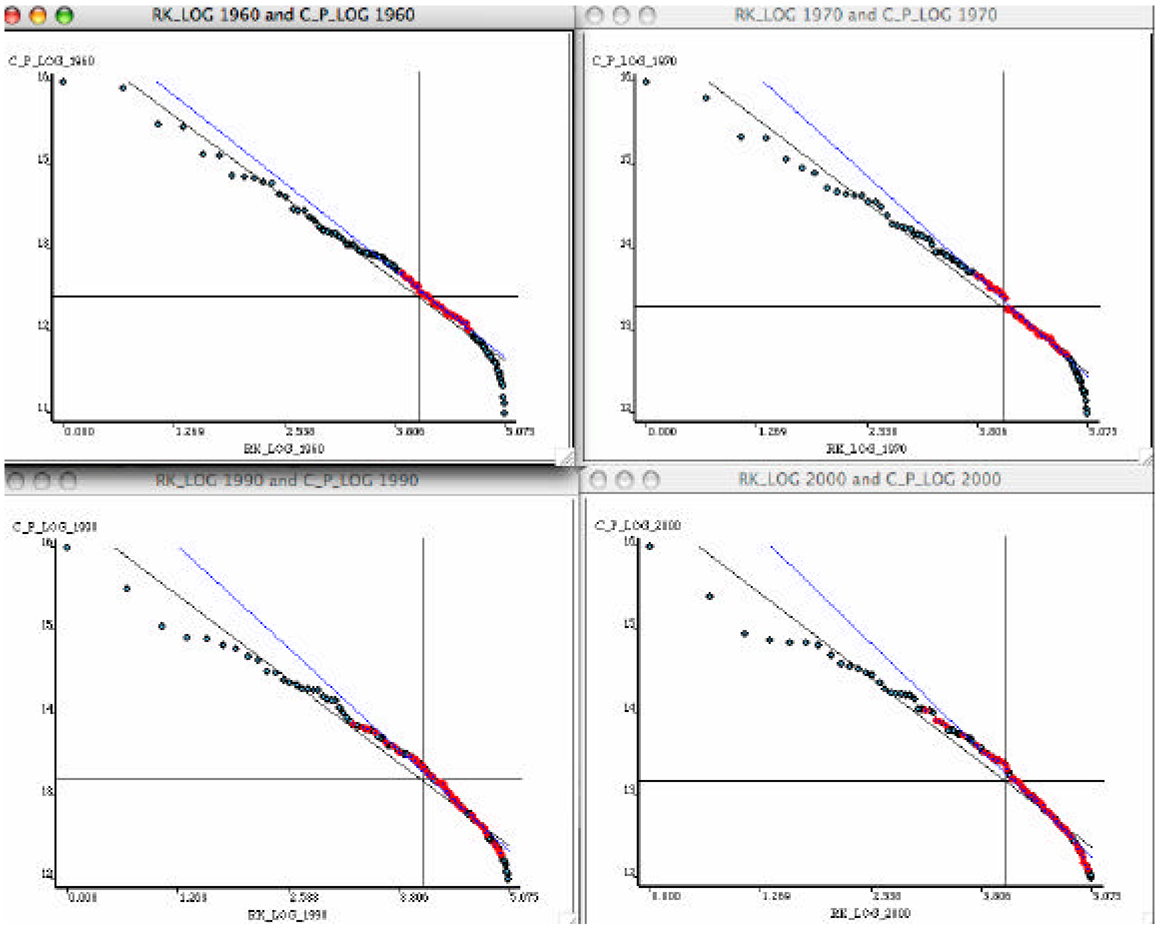


Figure 8. Middle Rank Dynamics from a Forward Perspective (1960-2000)

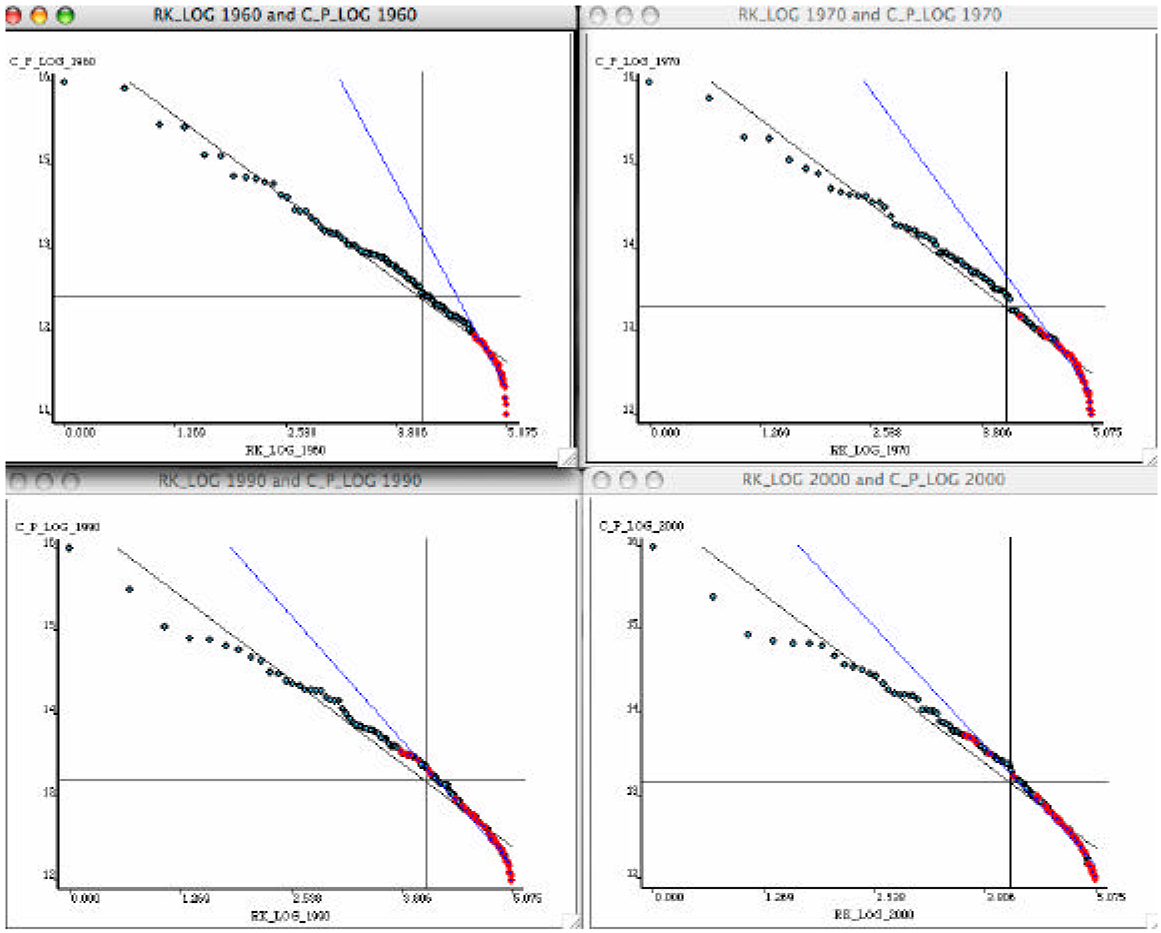


Figure 9. Lower Rank Dynamics from a Forward Perspective (1960-2000)

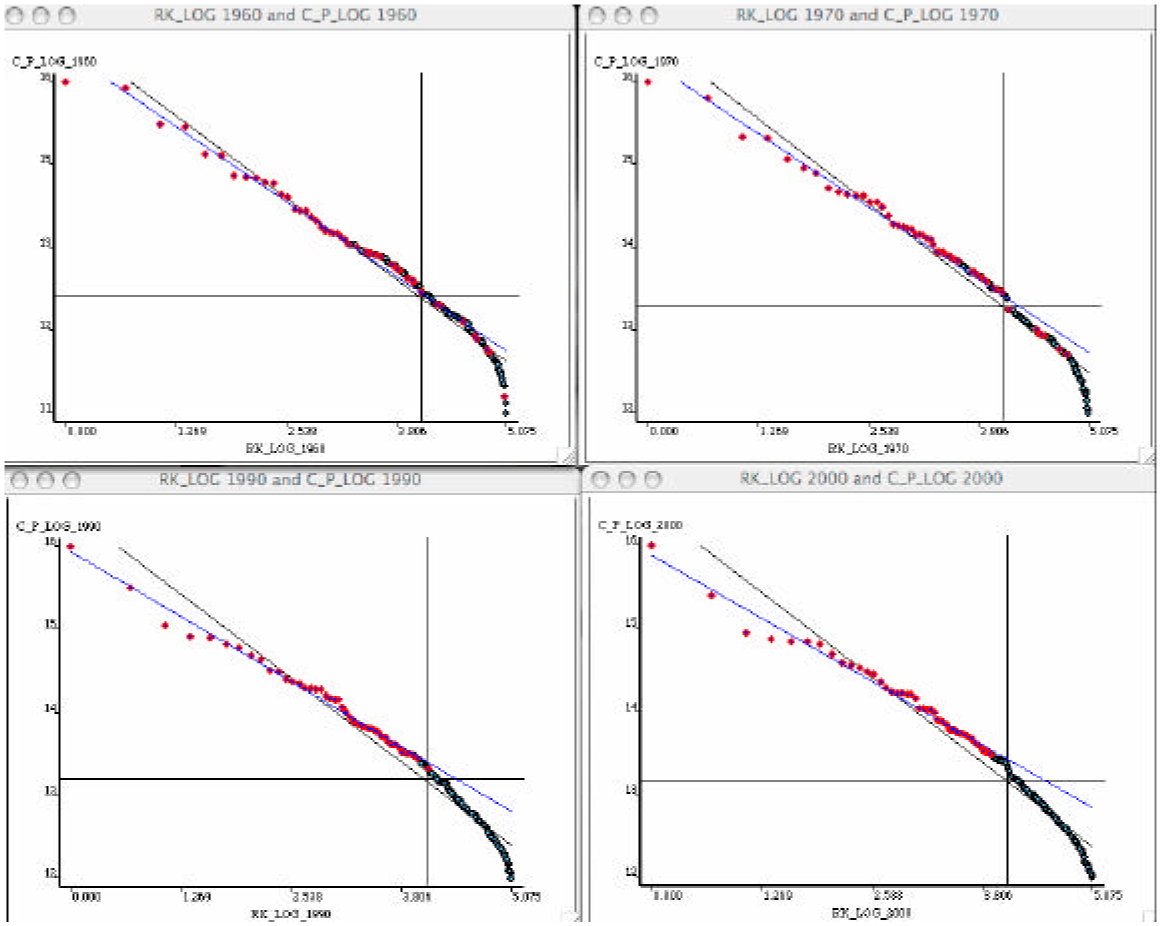


Figure 10. Upper Rank Dynamics from a Backward Perspective (2000-1960)

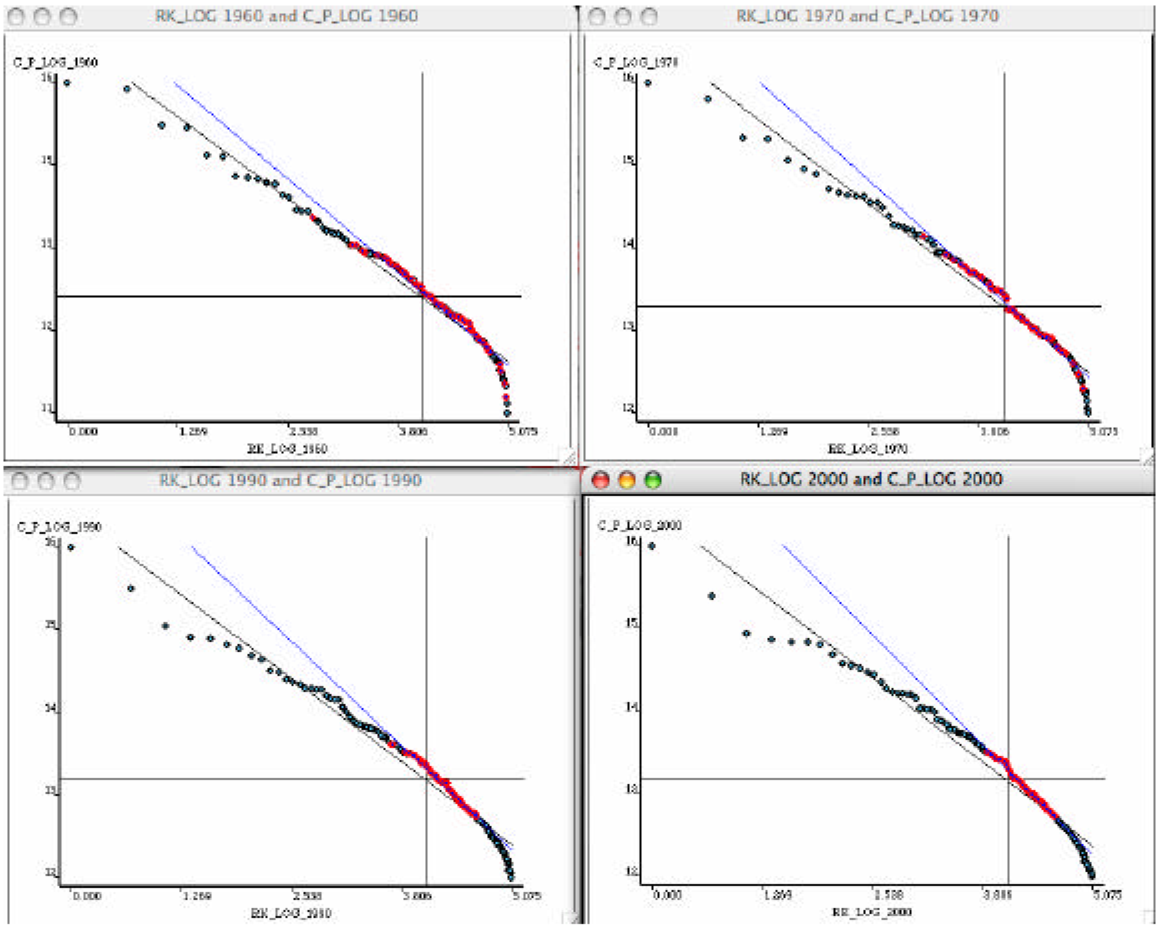


Figure 11. Middle Rank Dynamics from a Backward Perspective (2000-1960)

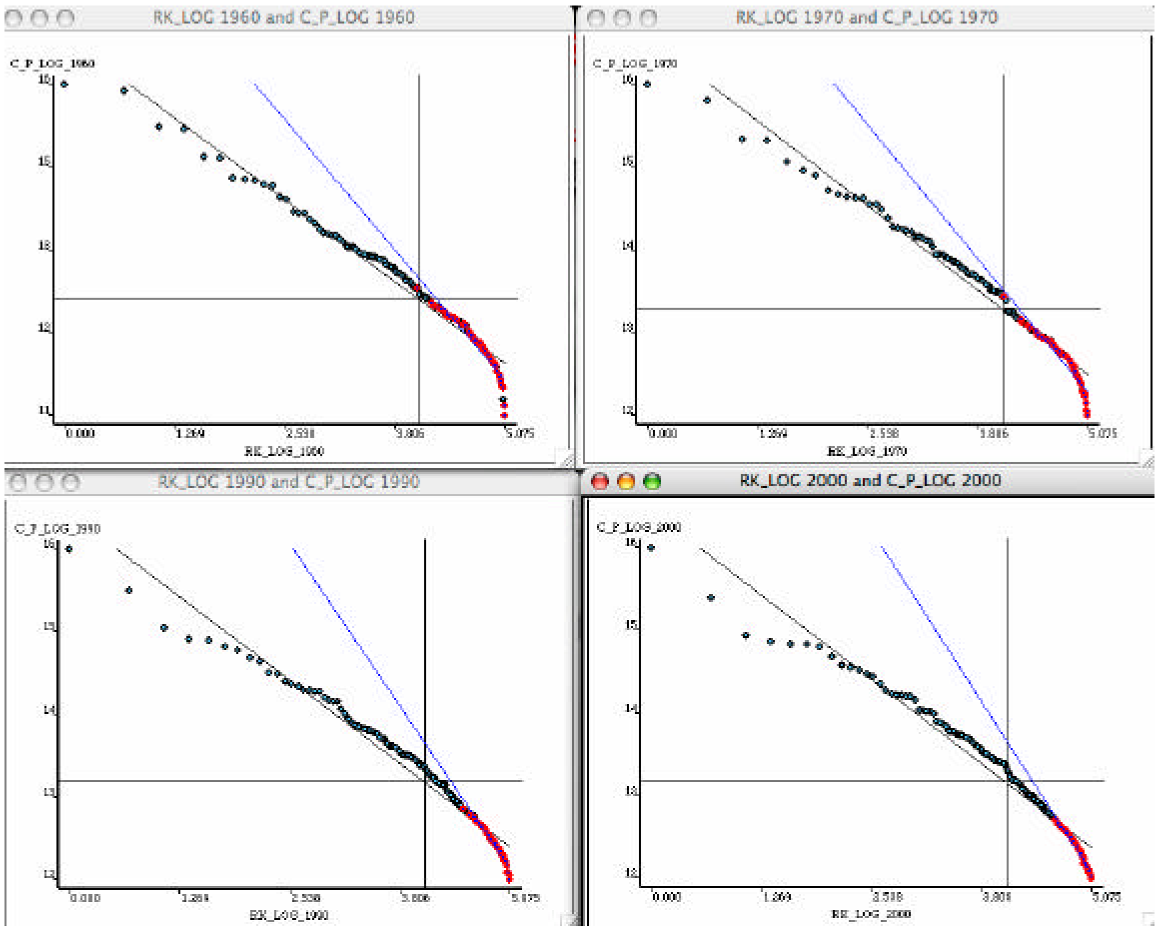


Figure 12. Lower Rank Dynamics from a Backward Perspective (2000-1960)

Eight blue colors are used in the right views in Figure 13 and Figure 14, with each color representing a BEA region on the right views (the larger the number representing BEA region, the darker the blue; for example, number Eight is Far West BEA region which includes California, Nevada, Oregon, Washington and the darkest blue points representing the cities located in this region have the most diverse development pattern in both inward and outward diffusion plots).

Space-Time Correlation is identified for Rocky Mountains BEA region (highlighted one on the left view) shows that region has a more obvious inward diffusion than outward diffusion (the former four points fit to the trend line better than the latter).

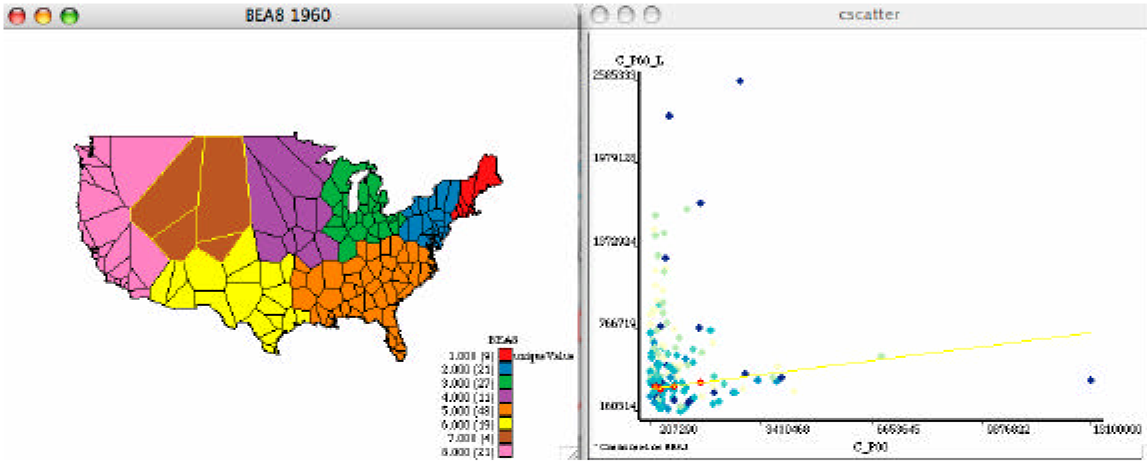


Figure 13. Inward diffusion in Rocky Mountain BEA Region, 1960-2000
Note: the right view depicts the correlation of urban size in 2000 at a location with the average for its neighbors in 1960. It can be consider as an inward diffusion (from the neighbors now to the core in the future) (Anselin 2005). It is a positive relationship in this case.

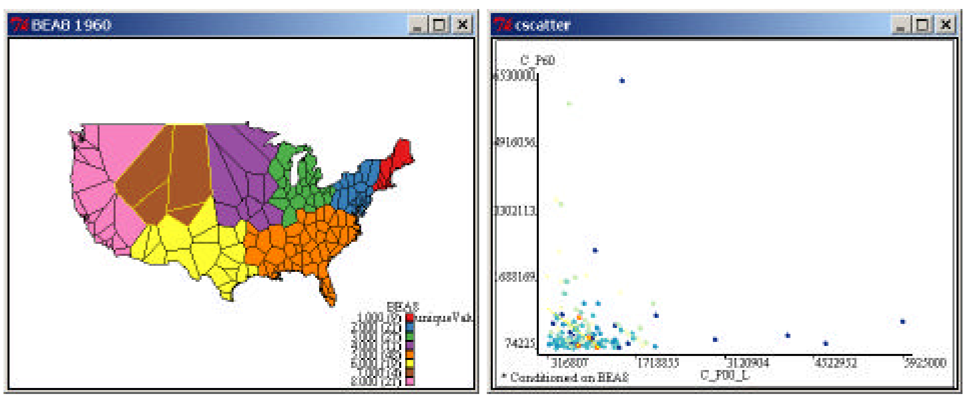


Figure 14. Outward diffusion in Rocky Mountain BEA Region, 1960-2000
Note: the right view depicts the correlation of urban size in 1960 at a location with the average for its neighbors in 2000. It can be consider as an outward diffusion (from the core now to its neighbors in the future). It is a negative relationship here.

2.3. Individual Spatial Dynamic

An urban system is considered as an evolving organization of city groups. Hence, Markov chain approaches can be used to study the evolution of the urban size distribution over time. Convergence classification based on each city's average rank movement uncovers the individual rank mobility and the degree of spatiotemporal convergence clustering. A red color means the former size is below the average and its average shift in the forty years is moving further down. An orange color means the former size is below the average and its average shift is moving up. A light yellow means the former size is around the average and its average shift is staying no change. A light green color means the former size is above the average but its average shift is moving down. A Dark green color means the former size is above the average and its average shift is moving up. If we group the dark green and orange ones as the winner of rank competition and the red and light green ones as the loser, there are two obvious clusters of rank gaining and losing areas with the former located in the southwestern U.S. and the latter in the Middle West.

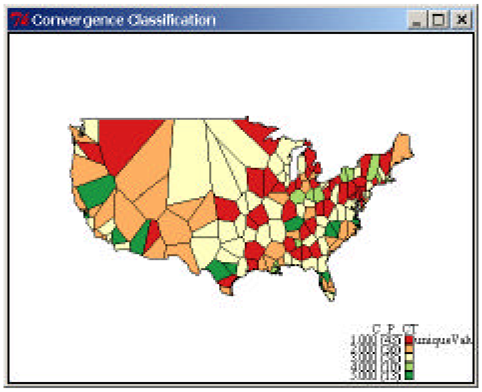


Figure 15. Convergence classifications (Average Change) based on Marchov Chain

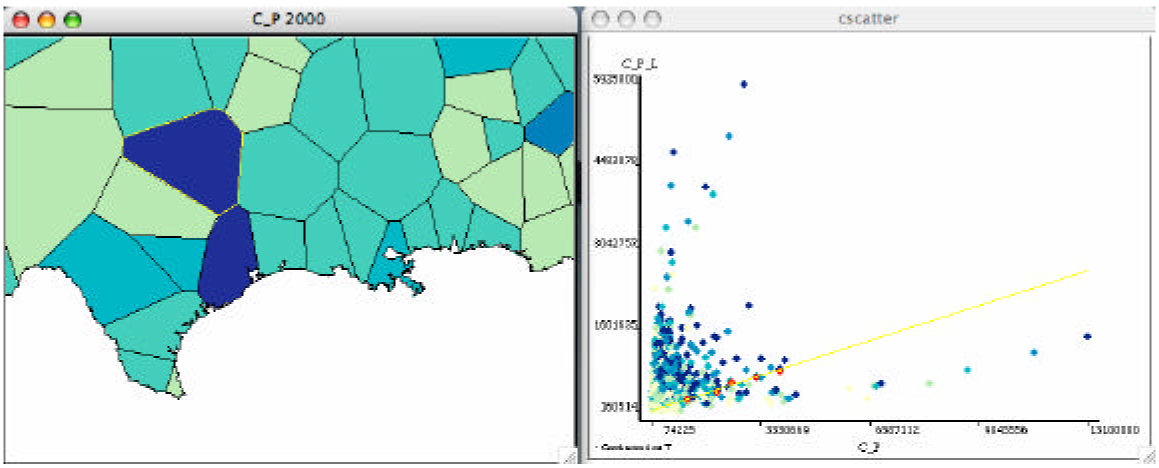


Figure 16.

The left view in Figure 16 is a zoomed-in map, where Dallas (the highlighted one) is located. A conditional scatter plot in the right view is used to combine the Moran scatter plots from each year in a single view. The observations on each urban size and that of its spatial lag are then conditioned on the five time points, and the conditioning uses color depth to indicate early (light color) versus more recent (dark color) observations. There are five different blue colors for the years and 800 points, which represent the 160 cities at each time point. The conditioning reveals that the dispersion in urban size has increased substantially over time in the two wings (leaning towards X axis as the Dallas case or leaning towards Y axis) and this can not be revealed by the traditional approaches because the existence of the majority similar changes (the clustered points in the middle between the two wings in the right view) dominate the overall indicators. When Dallas is selected on the right map, a link is triggered in the destination view (the right view). The spatial dynamic of Dallas and its vicinity is shown by the highlighted five points and a yellow trend line. This dynamic has been much different from the majority dynamics in the US urban system evolution. Compared to the Dallas case, Detroit and its vicinity has a slow development pattern (the five highlighted points are much more clustered in Figure 17).

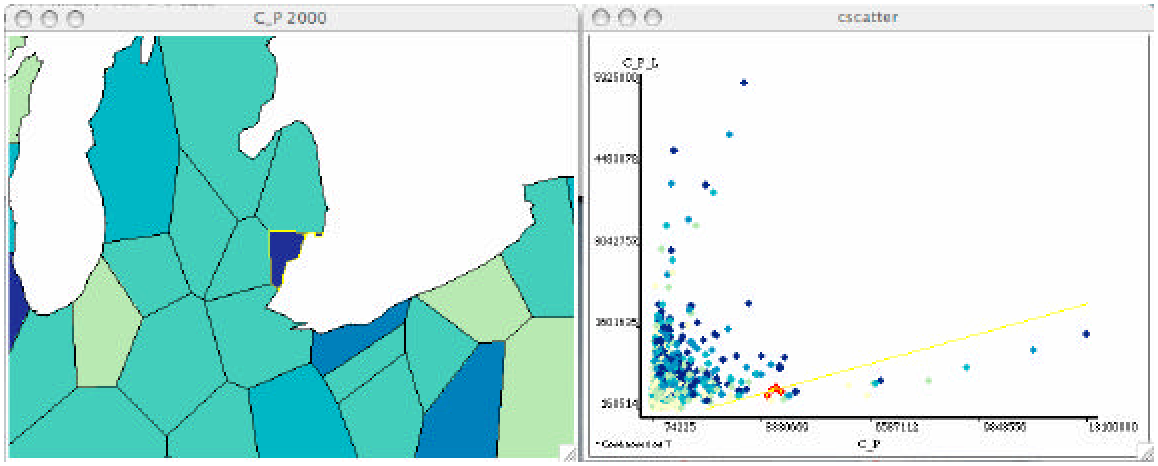


Figure 17.

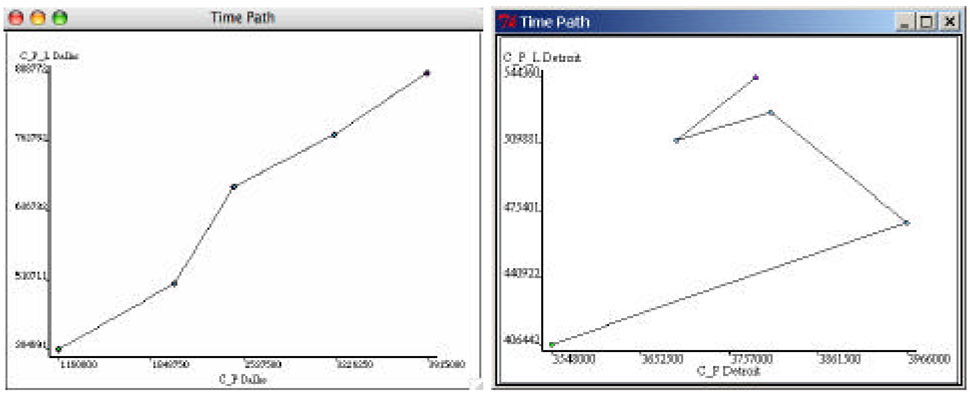


Figure 18. LISA Time-Path Plot

Figure 18 illustrates the pair-wise movement of a spatial unit's urban size and its average neighbor's over time. This view is helpful in identifying levels of stability of city across a given structural process. Individual city's time path can also be visualized to check the dynamic in a time series format, with the points taking the order from left to right, representing 1960, 1970, 1980, 1990 and 2000 (Figure 18). The left graph is Dallas and its average neighbor while the right one is the case of Detroit. Dallas grows very fast with its local networked cities while Detroit lost population after 1970 and its average neighbor's population stagnated for about two decades.

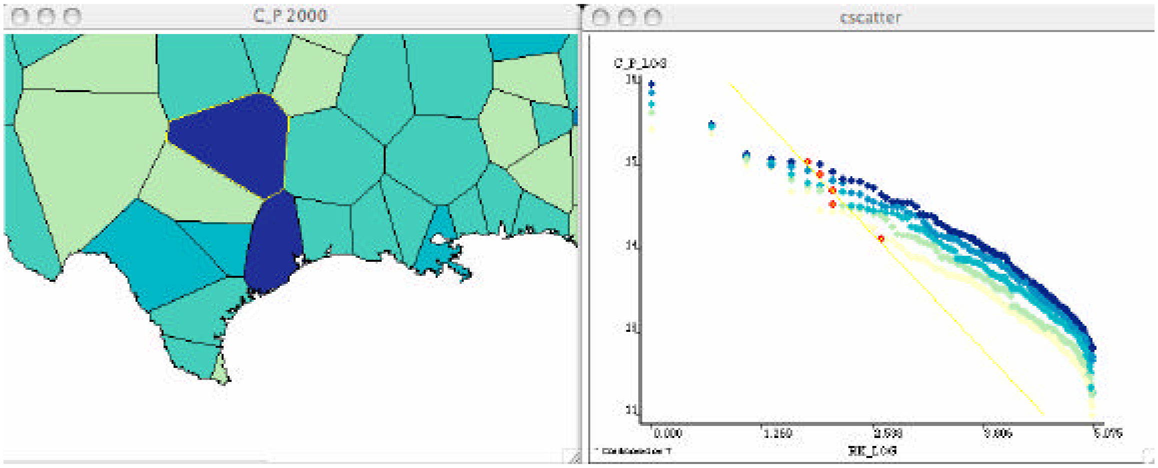
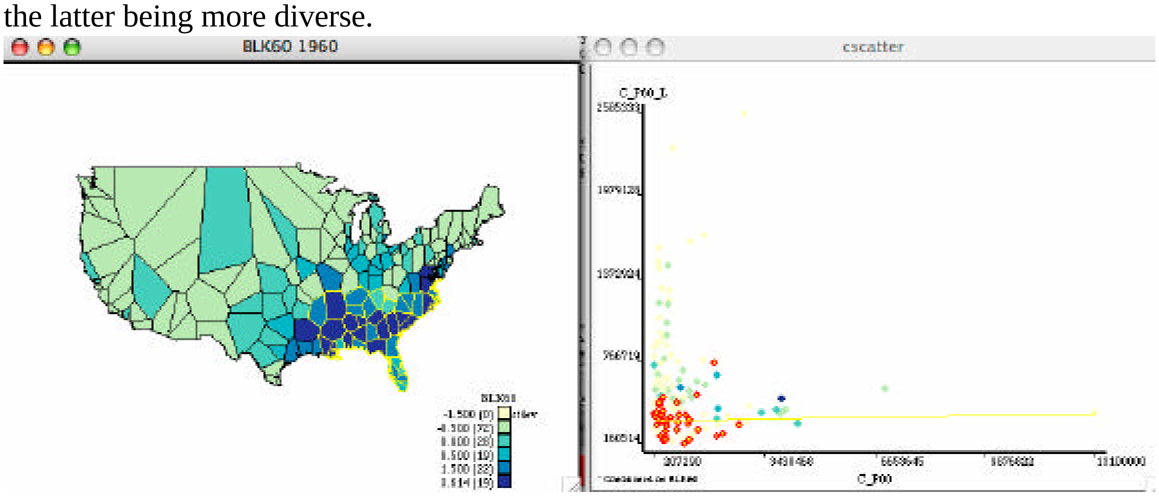


Figure 19.

The right view in Figure 19 combines the Zipf's plot at five time points into one map. The lightest color represents Year 1960 and the darkest one is Year 2000 with the rest three years between them, which suggest an overall expansion of urban size over years. When Dallas is chosen on the left map, its dynamic in the Zipf's plot is highlighted on the right map. There is a big rank jump between 1960 and 1970. The status stagnates between 1970 and 1980, then keeps moving up between 1980 and 2000.

2.4 Driving Forces Perspective

Urban system evolution is the outcome of many driving forces or factors. Figure 21 examines how the percentage of African American population in the beginning year 1960 might influence the inward diffusion of urban size evolution, that is, how the racial factor might functions during the period when the core (in Time T+T0) resist the influence from its neighbors (in Time T). Figure 21 shows the core's urban size evolution is quite different considering this driving force in the southeastern U.S. (high African American percentages) and the western U.S. (low African American percentages), with the latter being more diverse.



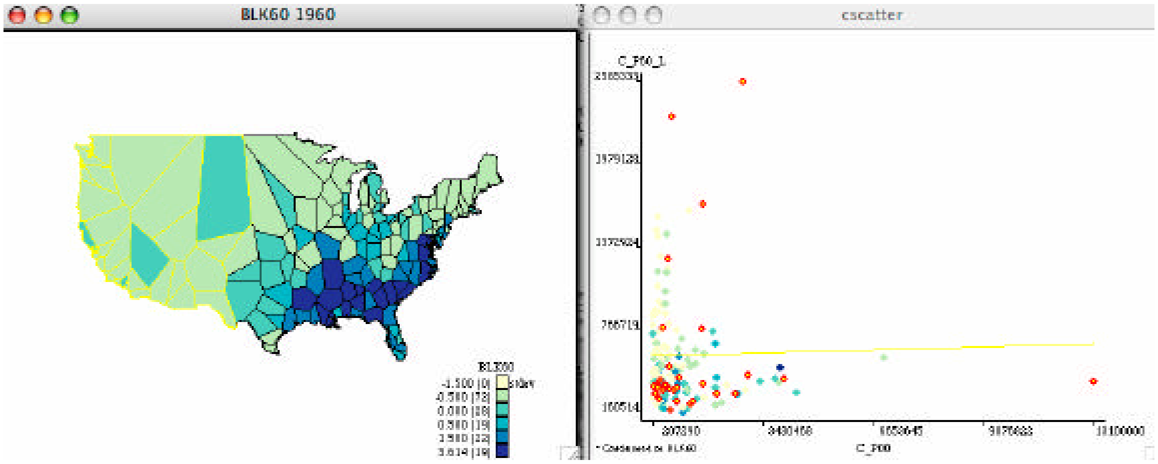


Figure 21. African American Percentage
Note: the left view is a standard deviation classification of African American percentage in 1960; the right view is the scatter plot of inward diffusion conditioned on the percentage of African American (the higher the value, the darker the color)

3. Geography Matters and Urban Emergence: Conclusions and Future Work

This research examines urban system evolution in the United States from space-time perspectives. It illustrates that these new approaches successfully capture spatial dependence in the urban system evolution, that is, geography matters in the urban growth. An important characteristic of the urban system emergence is the dynamic change of cities in rank-size distribution (Batty 2003). And our new mapping methods reveal the existence of dramatic individual dynamic and regional variance masked by the overall patterns of system evolution. From space, time and distribution perspectives, spatial competition among cities, regions and rank groups are visualized in an interactive way, which leads to insightful discoveries in the research questions interesting many urban and economic geographers (Beckmann 1958; Harris 2004).

While this space-time approach shows promise, much more work needs to be done on its theoretical properties and a number of implementation issues also need to be further investigated such as the definition of the spatial weights matrix for the cities and the dominant area of cities. Two other research works will be carried out, one is to do an investigation of urban system comparison across countries and another one is to further implement multivariate analysis of driving forces into urban size spatial dynamics studies.

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