A Clustering-Based Approach to a Special Capacitated Facility Location Problem

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Abstract:
This paper presents a clustering-based approach to address a special case of the Capacitated Facility Location Problem (CFLP). The goal is to find the optimal locations of a finite set of facilities to satisfy the demands of a large set of fixed locations, while the sum of the demands at the point locations associated with the same facility cannot exceed its capacity. We develop a clustering-based approach to this special CFLP instance and evaluate this approach with various experiments. The overall performance is evaluated using the average distance from the point locations to their assigned facilities. The experiments on data sets of random distribution and clustered distribution show that the clustering-based approach can lead to near-optimal configurations of facility locations with fast converges, no matter whether the capacity of facilities is sufficient or insufficient for the total demands. We also test the method with a real data set (i.e., allocating emergency shelters to serve people on a Caribbean island), where demands at point locations are not equal and the capacities of facilities are not the same.

Keywords:
Capacitated Facility Location Problem, clustering, K-means

1. Introduction

The allocation of facilities is a spatial decision problem that has various applications, e.g., emergency management, distribution logistics, telecommunication, marketing analysis, routine delivery system design, and urban and regional planning (França, M. Sosa et al. 1999; Baldacci, Hadjiconstantinou et al. 2002; Correa, Steiner et al. 2004; Higgins and Laredo 2006; Negreiros and Palhano 2006; Wu, Zhang et al. 2006). For example, we may need to allocate a certain number of emergency shelters to protect local people during possible natural disasters. The shelters should be accessible to people as much as possible. A solution to this shelter allocation problem consists of two parts: the locations of the shelters and the people residency that is covered by each individual shelter. The overall accessibility of all shelters can be quantified (or evaluated) by the average distance from the residency to their designated shelters. Moreover, the total capacity of the shelters may not be sufficient to serve all the people. Such facility location problems can be formulated as a variation of *Capacitated Facility Location Problem* (CFLP), as stated in equation 1.1.

$$V(S) = \frac{\sum_{j=1}^{K}\sum_{i=1}^{n_j} p_j^{(i)} \left| X_j^{(i)} - L_j \right|}{\sum_{j=1}^{K}\sum_{i=1}^{n_j} p_j^{(i)}}, \text{ where } \sum_{i=1}^{n_j} p_j^{(i)} \le C_j, \quad 1 \le j \le K, \tag{1.1}$$

$V(S)$ represents the objective function of a solution S. K is the total number of facilities and n_j is the number of point locations served by facility j. $p_j^{(i)}$ is the population of the i^{th} point location assigned to facility j. $X_j^{(i)}$ is the i^{th} point location assigned to facility j and L_j is the location of facility j. $\left|X_j^{(i)} - L_j\right|$ represents the Euclidean distance between the i^{th} point location and its designated facility j. The primary constraint is that the total population covered by a facility cannot exceed its capacity S_j. The goal is to find the optimal (or near-optimal) locations of a finite set of facilities and decide the coverage of each facility (Correa, Steiner et al. 2004; Ceselli and Righini 2005)

While a classical CFLP problem forces all point locations to be served and allows point locations to be served by one or multiple facilities (Nauss 1978; Chudak and Williamson 2005), in the research reported here, one point location can only be served by one facility. This research transforms the clustering concept to formulate solutions for location-allocation problems. A clustering-based approach is proposed to achieve a near-optimal solution to this location allocation problem. The remainder of the paper is organized as follows. Section 2 reviews relevant research works. Section 3 introduces the proposed clustering-based approach. Section 4 presents several experiment results with different datasets. Section 5 provides a summary of the research and a brief list of future works.

2. Related work

The facility allocation problem has many variations in various contexts, including CFLP, capacitated p-means problem (CPMP), capacitated centered clustering problem (CCCP), capacitated clustering problem (CCP). CPMP, CCCP, and CCP are similar to the problem defined in the previous section in that: (1) each point location is served by one facility; (2) the cost is modeled with dissimilarity measures or distances; and (3) the facility setup cost is ignored. However, while CPMP, CCCP, and CCP require that every point location should be allocated to at least one facility, our approach allows uncovered locations if the total capacity is insufficient.

Substantial research work has been carried out to address facility location problems (França, M. Sosa et al. 1999; Wu, Zhang et al. 2006). In general, existing CFLP approaches can be classified into three types: heuristic method, genetic algorithms, and exact algorithms. Heuristic methods include Lagrangian relaxation, bionomic metaheuristic (Maniezzo, Mingozzi et al. 1998), Forgy's method (Negreiros and Palhano 2006), column generation (Lorena and Senne 2004), scatter search (Diaz and Fernandez 2005), tabu search (França, M. Sosa et al. 1999), and variable neighborhood search (Negreiros and Palhano 2006). Genetic algorithms transform the optimization problem to a simplified representation (e.g., a sequence of bits) and a simple set of operations (e.g., production, mutation and crossover) (Lorena and Furtado 2001; Correa, Steiner et al. 2004; Lorena and Senne 2004). Exact algorithms include set partitioning technique (Baldacci, Hadjiconstantinou et al. 2002), branch-and-price algorithms (Ceselli and Righini 2005), and mixed integer programming models (Wu, Zhang et al. 2006). Neither heuristic methods nor genetic algorithms guarantee to find the optimal (or even near-optimal) solutions. Although exact algorithms are able to find optimal solution, they are often too computationally intensive and time consuming to be applied even to a relatively small data set (Ceselli and Righini 2005).

This research transforms the clustering concept to formulate solutions for location-allocation problems. Clustering analysis is useful in many contexts, e.g., pattern discovery, document retrieval, image segmentation, etc. The primary task for a clustering method is to group a given collection of unlabeled observations, data items, or feature vectors, into a set of meaningful clusters (Jain, Murty et al. 1999). Because the grouping/labeling does not depend on prior knowledge or training data, clustering methods are data driven and can discover natural groupings of data items (Jain and Dubes 1988; Jain, Murty et al. 1999; Han, Kamber et al. 2001; Guo, Peuquet et al. 2003). Clustering analysis can be distance-based, model-based, and density based (Jain, Murty et al. 1999; Guo, Peuquet et al. 2003). The method presented in this paper is distance-based because distance is the primary factor for the accessibility of facilities and thus for the allocation of facilities.

Specifically, the method we propose is similar to the k-means clustering (Duda, Hart et al. 2001). K-means is a partitional clustering approach that creates a single partition of the data items instead of a hierarchical dendrogram. It divides the dataset into non-overlapping groups that cover the whole data space. The k-means clustering method assumes that the number of clusters is known. It consists of three steps (Jain, Murty et al. 1999): (1) randomly choosing k cluster centers within the space of the dataset; (2) assigning each data item (presented by a point in a 2D or higher dimensional space) to the closest cluster center; (3) recalculating the cluster centers using the points assigned to each cluster. Steps 2 and 3 are repeated until the result converges.

3. A Clustering-Based Approach

Suppose there are n demand points and p facility locations. The capacity for each facility is c_i (i=1..p) and the demand of each demand point is d_j (j=1..n). The demand points are of some distribution, which can be clustered, random, or a real-world situation (e.g., population grids of a region). The location of each demand point is fixed while the facility locations may be reconfigured to minimize the overall cost. This research defines the cost as the average distance between each demand point and its designated facility. To simplify the presentation of the proposed approach, we start with a simplified scenario, which assumes that the total capacity of the p facilities is large enough to satisfy all the demands and that the capacity for each facility is the same. Following this simplified scenario, we will address complicated scenarios that the total capacity may be insufficient to accommodate all demands.

3.1. Sufficient Capacity

In cases where the total capacity of the p facilities is larger or equal to the total demands, the clustering-based method can be conceptualized as shown in Figure 1 to seek an optimal (or near-optimal) configuration of the p facility locations.

The three steps outlined in Figure 1 are similar to the three steps for the k-means clustering as briefly introduced in the previous section. However, both step (i) and step (ii), i.e., the allocation and relocation steps, are changed to fit the location allocation problem. Specifically, a normal k-means clustering method assigns a point to its nearest cluster while the method introduced here considers the capacity of each facility. Moreover, the recalculation of facility locations (step ii—relocation) may vary depending on the cost functions of applications. Since in this research we minimize the average Euclidean distance between demand points and their corresponding facilities, the new location of a facility is the centroid of all the demand points that are currently assigned to that facility. Step (iii), i.e., the iteration step, compares the newly calculated coordinates with the previous coordinates of each facility. If there is no significant difference between the two, the algorithm is terminated. Through such an iterative refining process, the method can quickly adapt the locations of facilities to the distribution of demand points (see Figures 4-6).

(i) **Allocation:** Given a configuration of the *p* facility locations, assign each demand point to a facility so that (a) each demand point is assigned to its nearest facility if its capacity allows, and (b) if two demand points compete for the same facility, the one with a shorter distance has the higher priority, and (c) if a demand point is rejected by its nearest facility, it will compete for its second nearest facility and so on until it finds a facility that accepts it.
(ii) **Relocation**: Recalculate the location of each facility based on the locations of all the demand points that are assigned to that facility.
(iii) **Iteration**: Repeat above steps until the result converges.

Figure 1: An outline of the clustering-based method to the capacitated location allocation problem, when each facility has the same capacity and the total capacity of facilities exceeds the total demands.

Figure 2 shows the detailed algorithm for the allocation step, i.e., step (i) in Figure 1, which allocates demand points to facilities with capacity considerations.

1. Form a location pair between each facility and every demand point;
2. Sort all the location pairs based on their distances in an ascending order, thus the shortest pair first;
3. Start from the first (i.e., shortest) pair and assign the demand point of that pair to the facility in that pair, unless the facility is full. If the demand point associated with the pair is assigned to the facility, (1) the available capacity of the facility is decreased by the demand of the point, and (2) this demand point cannot be served by other facilities.
4. Repeat step 3 until the last location pair in that sorted order. By then, each demand point shall have a facility associated with it.

Figure 2: The location allocation algorithm for the step (i) in Figure 1.

3.2. Insufficient Capacity

In cases of insufficient capacity, it is not realistic to satisfy the demands at all point locations. A certain number of point locations must be left unserved. The result of the clustering-based method can be sensitive to the initiation of facility locations due to the insufficiency of capacity. Figure 3 presents an extended version of the clustering-based method for CFLP problems with insufficient facility capacity.

As shown in Figure 3, the method for insufficiently capacitated CFLP cases differs from the method for sufficiently capacitated CFLP in that it involves two more steps: step (i) and step (v). Specifically, step (i) forces the allocation to start from a sufficient-capacity case to satisfy the total demand. Once the result converges, the capacity for each facility is gradually reduced to its actual capacity while the iteration process proceeds. This process continues until the capacities of facilities are reduced to their original amounts and the result converges.

(i) **Enlargement**: Increase the capacity of the facilities so that the total of the capacity is larger than the overall demand.
(ii) **Allocation:** Given a configuration of the p facility locations, assign each demand point to a facility so that (a) each demand point is assigned to its nearest facility if its capacity allows, and (b) if two demand points compete for the same facility, the one with a shorter distance has the higher priority, and (c) if a demand point is rejected by its nearest facility, it will compete for its second nearest facility and so on until it finds a facility that accepts it.
(iii) **Relocation**: Recalculate the location of each facility based on the locations of all the demand points that are assigned to that facility.
(iv) **Iteration**: Repeat from step (ii) until the result converges.
(v) **Reduction**: Decrease the capacity of the facilities, repeat from step (ii) until the capacity of the facilities equal to the original amount.

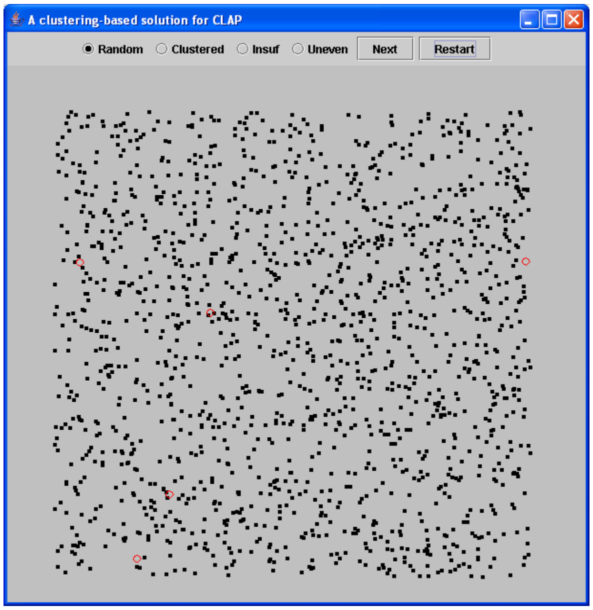
Figure 3: An outline of the clustering-based method to the capacitated location allocation problem, when each facility has the same capacity and the total capacity of facilities is less than the total demands.

4. Evaluation Experiments

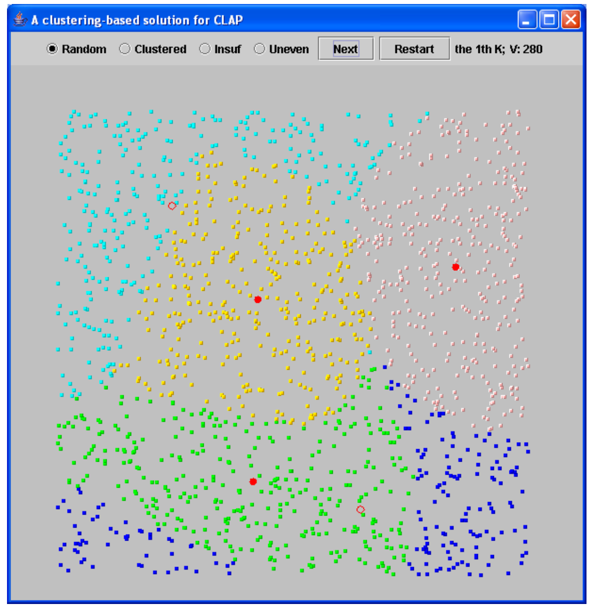
4.1. Experiments with Random Data

We carried out several experiments with different data sets of various distributions. Figure 4 presents the optimization process of a point data set of a random distribution. The data set consists of 1,500 points and 5 facilities. Each point location has a demand (population) of 1 and each facility has a capacity of 345. The demand value for each point location can be different. Here they are set to the same value to simplify the presentation and allow visual verification of the result. In Figure 4, red dots represent facilities and points in other colors represent demand points. Points associated with the same facility are in the same color. Facilities that are filled to their capacity are represented by filled circles and facilities that are not full and can serve more point locations are represented by empty circles.

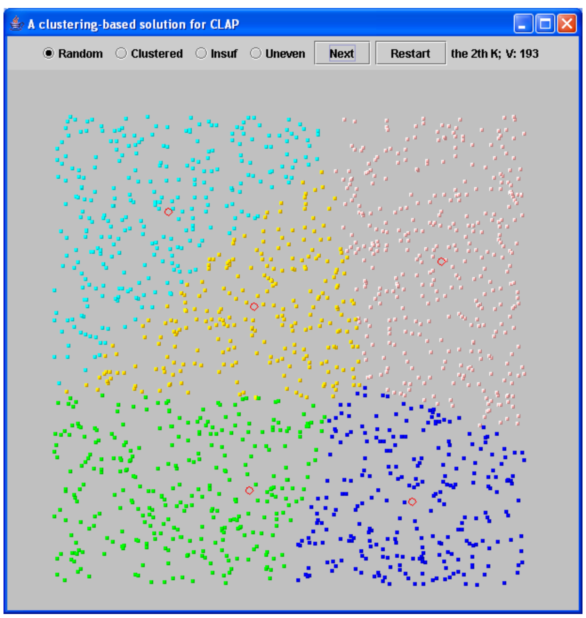
Figure 4 shows how the clustering-based method finds the optimal locations of facilities. The initial locations of the five facilities are randomly chosen (Figure 4-a). Demand points are not assigned to their nearest cluster centers as shown in Figure 4-b and Figure 4-c since these figures show the configuration before the relocation. Instead, facilities are shown as the cluster centroids of the member point locations. That is why the boundary between the clusters in yellow and in cyan is much closer to the center of cluster in cyan. The centroids of clusters in blue and green are inside the same cluster (the cluster in green). The cluster in blue is divided into two parts by the cluster in green (Figure 4-b). The average distance at this stage is 280. But a much more reasonably solution is achieved in the next allocation-relocation (Figure 4-c), during which the cluster in green shifts to the bottom left corner. As the configuration converges, the five facilities are positioned at the centroids of their associated clusters (Figure 4-d). It is noted that the boundary between the cyan cluster and the yellow cluster appears in the middle of the two centroids. Further, the average distance of point locations to their designated facilities decreases from 280 to 182.



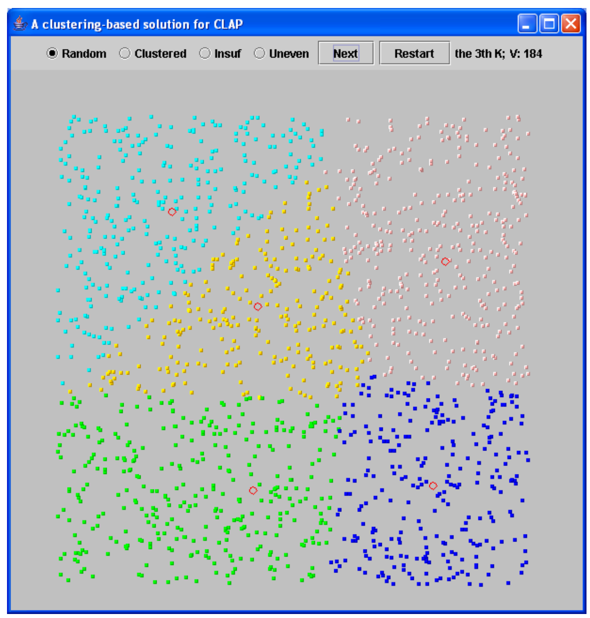
(a).Initial facility locations



(b). After the 1st relocation (V:280)



(c).After the 2nd relocation (V:193)

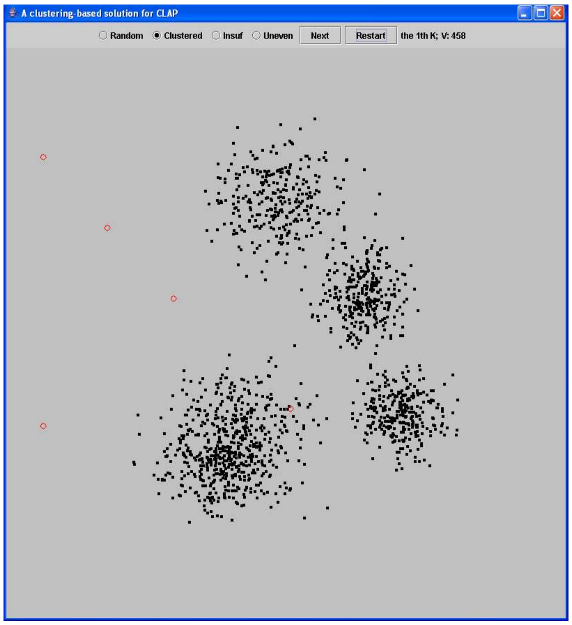


(d). After the 5th relocation (V:182, final)

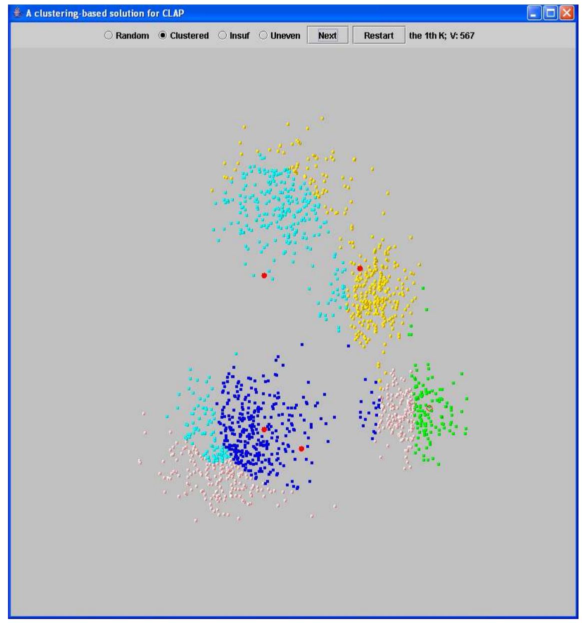
Figure 4: The optimization process of sufficiently capacitated facilities and randomly distributed point locations.

4.2. Experiments with Clustered Data

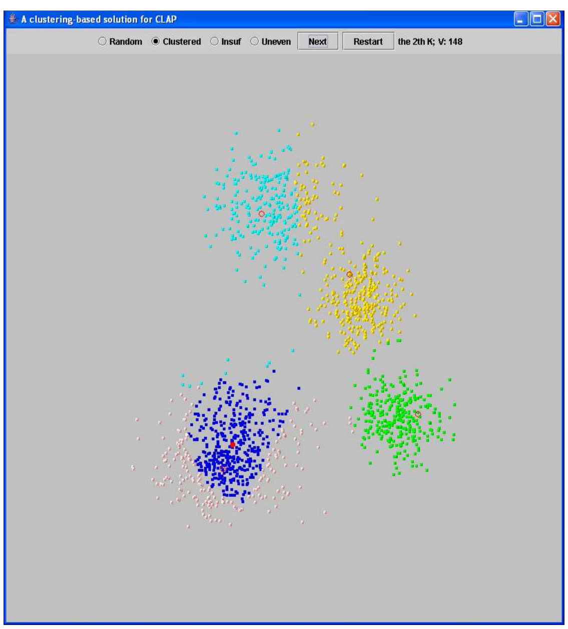
Figure 5 shows the allocation-relocation optimization process with a data set of a clustered distribution. The dataset consists of five clusters, each containing 300 points. The demand at each point location is 1 and the capacity of each facility is 345. Red dots represent facilities. Facilities that cannot serve more point locations are represented by filled circles while facilities that can serve more point locations are represented by empty circles. Points in other colors represent demand points. Demand points that are assigned to the same facility are painted in the same color.



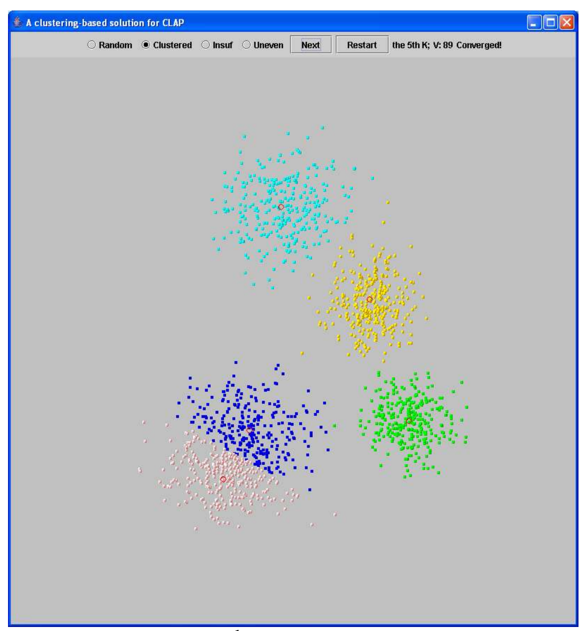
(a). Initial facility locations



(b).After the 1st relocation (*V*: 567)



(c). After the 2nd relocation (*V*: 148)



(d). After the 4th relocation(*V*:93,final)

Figure 5. The optimization process for sufficiently capacitated facilities and clustered point locations

The five facilities initially scatter around in the left half of the area (Figure 5-a). During the first relocation (Figure 5-b), the facilities are pulled to the midst of points. During the second allocation-relocation (Figure 5-c), the cluster is green and its centroid are firstly identified. The average distance decreases sharply from 567 to 148. As the procedures terminates, three isolated clusters of points (cyan, yellow and green) and their associated centroids are found (Figure 5-d). The boundary of the remaining two clusters (in blue and pink) which are connected is delineated (Figure 5-d). The average distance is reduced to 93.

4.3. Experiments with Insufficient Capacity

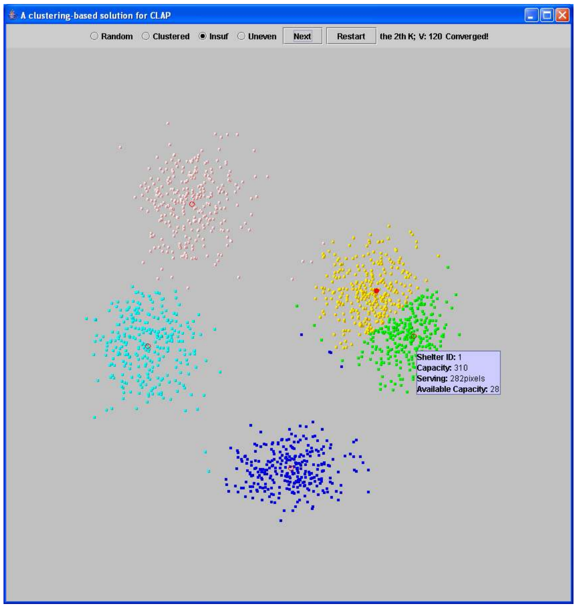
For cases that the total capacity is insufficient to meet all demands, we only use a clustered data distribution to evaluate our method because it is difficult to visually detect possible problems of the method if the demand points are randomly distributed.

Figure 6 shows the optimization process to find the optimal locations of five facilities for a data set consisting of five clusters, each containing 300 points. The demand at each point location is 1 and the capacity of each facility is 225. The search for optimal facility locations starts from an increased capacity of 345 for each facility. Two clusters of points are connected in the middle right portion of the data space while the other three clusters are separated (Figure 6-a). The first converge is attained with an average distance of 492 (Figure 6-a). Two points, which are closer to the center of the cluster in a cyan color, are assigned to the cluster in blue. All facilities are fully occupied except the one for which the member points are in green. The second round of allocation-relocation starts with a reduced capacity (Figure 6-b, the capacity is reduced by 10% for each round of allocation-relocation). On the other hand, the cluster in green recruits more points and fewer point locations are allocated to three other facilities.

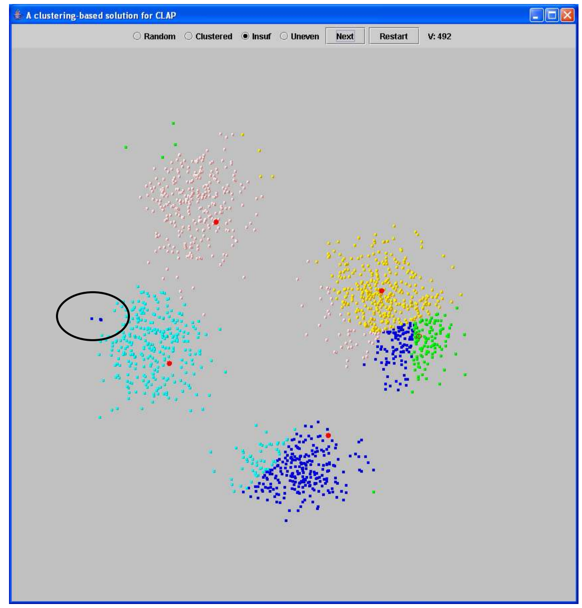
From the third allocation-location (Figure 6-c), the overall capacity (1,395) becomes insufficient to satisfy the overall demand (1,500) of the point locations. All the facilities serve point locations to their maxima and the average distance of member points to the serving facilities is 87 (Figure 6-c). The clusters contain points closest to them while the scattered points around the fringe of the clusters cannot be served. At the last round of allocation-relocation (Figure 6-d), the capacities of facilities change to the original value (225).The locations of the cluster centroids do not change significantly while the clusters are more compact than in the last convergence. The final configuration solution is achieved with an average distance of 73.

4.4. Experiments with Real Data

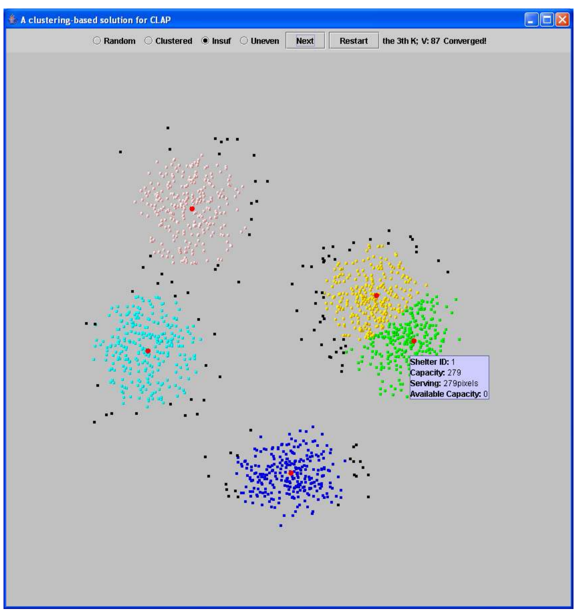
This section presents an experiment with a real-world application, which is to re-position 53 emergency shelters for the St. Vincent island off the coast of Venezuela. St. Vincent has a 5-10% annual chance of being struck by a hurricane (Pielke, Rubiera et al. 2003). For such a Small Island Developing States (SIDS), local residents have limited financial means to evacuate the island. Thus, emergency shelters are essential for providing people refuges from hazards and reducing vulnerability. Naturally, a network of emergency shelters needs to consider the population distribution to be easily accessed by local residents. The census data for each enumeration district are converted to population figures for equal-sized grids (20 meter by 20 meter). 45,027 population grids are created and the total population is 98,111. The capacity of emergency shelters ranges from 37 to 395 and the total is 8,734.



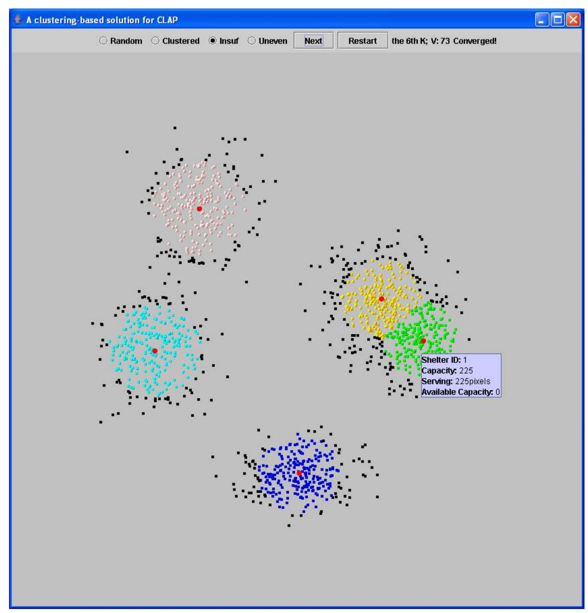
(a) After the 1st round of relocations (V:492)



(b) After the 2nd round of relocations (V:120)

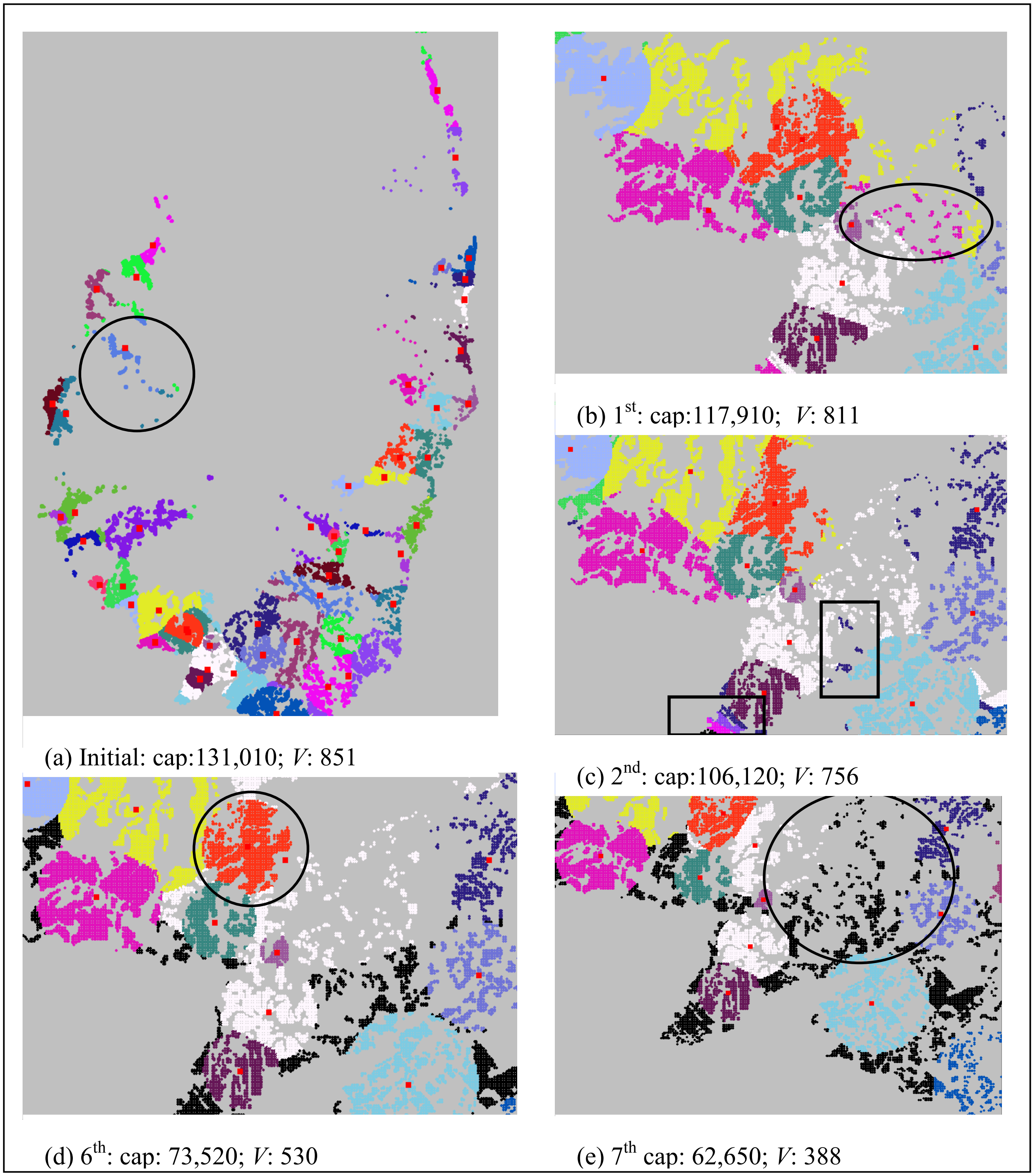


(c) After the 3rd round of relocations (V:87)



(d) After the 4th round of relocations (V:73)

Figure 6: The allocation process of insufficiently capacitated facilities and clustered point



(a) Initial: cap:131,010; V: 851

(b) 1st: cap:117,910; V: 811

(c) 2nd: cap:106,120; V: 756

(d) 6th: cap: 73,520; V: 530

(e) 7th cap: 62,650; V: 388

Figure 7. The allocation process with 53 emergency shelters and a large set of population grids for the St. Vincent island off the coast of Venezuela.

For demonstration purpose, the capacities of all shelters are magnified by a factor of 7 so that the total capacity is much larger than the original but still less than the total demands. In Figure 7, red squares represent emergency shelters while rectangles of a much smaller size represent population grids. Figure 7-b through Figure 7-e show the lower left half of the island to reveal more details. Figure 7-a shows a configuration of the existing facilities and the allocation of population grids with the capacity magnified to be sufficient. Most people live along the coast and some shelters serve elongated areas. Figure 7-b shows the configuration with the first reduced capacity. Clusters in magenta and yellow are both discontinuous and include substantial distant points. As the capacities become even smaller, the clusters tend to be more compact (Figure 7-c). Distant points that are separated from the majority of the clusters in yellow and magenta (circled in Figure 7-b) are taken by the facility which serves the point locations in white. The total capacity becomes insufficient from the 6^{th} round of allocation-location (Figure 7-d). The enclaves in dark blue surrounded by point locations in white and blue are left uncovered, as well as the point locations to the lower left of the cluster in brown. compared to previous configurations, the serving area in orange is apparently more compact. Figure 7-e shows the final configuration after the 7^{th} round of allocation-location. As a whole, the area covered by facilities includes most populated area. And the area associated with an individual facility tends to be circle-shaped. Sparsely populated area in the upper right half is uncovered.

5. Discussion and Conclusion

This paper presents a clustering-based approach to a special version of CFLP with one serving facility constraint and consideration of insufficient capacity. We transform the k-means clustering strategy to address location allocation problems. Specifically, each facility first satisfies its closest demand points unless the points are even closer to another facility that is not full yet. It also ensures that each demand point is served by its closest facility unless other demand points are even closer to the facility such that the facility cannot serve more demand points.

Evaluation results indicate our clustering-based approach provides a fast and efficient solution to the special CFLP. In the experiments with data sets of random distribution and clustering distribution, it take only 5 and 4 allocation-location loops to find the near-optimal locations of facilities. The quick convergence indicates this clustering-based approach is computational sparing, although the number of loops in the searching for the optimal configuration of facilities sure relates to the particular distribution of data set, the number of facilities, and the initial configuration. Moreover, the average distance constantly declines during the optimization process. This means solutions provided by this method tend to more efficient during the optimization process. Another feature of this clustering-based method is its conceptual simplicity.

The prototype of this clustering-based method is extended to handle cases where facilities are insufficiently capacitated. Experiment results show the adapted clustering-method can identify isolated clusters and locates facilities at the centroid of the dense area of demand points. We also tested the method with a real data set (i.e., allocating emergency shelters to serve people on a Caribbean island), where demands at point locations are not equal and the capacities of facilities are not the same. Future works will extend the method to address constraints in addition to the capacity limit and to deal with scenarios when the facility capacities are not the same.

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