Geographical Information Science to Enhance Location Coverage Modeling

Daoqin Tong

Alan T. Murray

Center for Urban and Regional Analysis
and Department of Geography
The Ohio State University
1036 Derby Hall
154 North Oval Mall
Columbus, OH 43210
USA

(Email: tong.45@osu.edu; murray.308@osu.edu)

Geographical Information Science to Enhance Location Coverage Modeling

Abstract

The use of points to represent facilities, demand or a region of interest in location modeling may not effectively address the geometrically complex shape of abstracted spatial features resulting in certain measurement errors. While point-based representations facilitate model construction, simplify data requirements and enable more efficient problem solution, advances in GIScience and spatial data acquisition highlight deficiencies in using overly rudimentary feature depictions, like points, in certain modeling instances. In this paper other representations, such as lines and polygons, are investigated in covering location problems. The evaluation of warning siren siting in the city of Dublin, Ohio is used to discuss nuances of point representation and other geometric shapes in location modeling. We illustrate how geometric sophistication and reality can be addressed in location coverage modeling through the use of GIS. Application results show that complex geometric shapes can be considered in site/service selection, and doing so in a way that remains computationally feasible.

1. Introduction

"Location" is an important factor for people to conduct certain activities. Many organizations face locational decisions. In the private sector such choices affect the ability of a firm to make use of capital investments and compete in the market place. In the public sector such choices involve not only monetary costs but social benefits (White 1979). As a result, location analyses assisted by planning and design processes are important in both the public and private sectors (Murray 2003).

Of interest in this paper are covering location problems, which measure effectiveness through assessment of whether demand can receive service. Often, service is considered adequate if the customer is within a certain distance or travel time of a facility. Coverage problems have been used in fire station location (ReVelle et al. 1989), bus stop design (Gleason 1975), warning siren siting (Current and O'Kelly 1992) and weather monitoring (Minciardi et al. 2003), among others. The objective of those problems is to find an optimal configuration of facilities to provide suitable service to demand.

To reach a solution to a location decision problem, a location model is often used (Birkin et al. 1996). Location models typically use points to represent facilities, demand or a region of interest (Miller 1996; Church 1999; Murray et al. 2006). A demand point can be a sub-area of a region, such as a city or a town, with associated aggregate information (Current and O'Kelly 1992). The primary reason to use a point instead of a geometrically complex shape (e.g., polyline, polygon, etc.) is that the point representation can simplify data handling and solution operations, and is easier for model construction. However, the real world cannot be simplified to points in certain situations. For instance, demand, such as census tracts or planning districts, or facilities, like parks or retail outlets, are often some areal unit (Murray 2003).

The issue related to how to represent geographic space, especially regional demand, in location modeling has received considerable attention in recent years (e.g., Miller 1996; Murray and O'Kelly 2002; Murray 2003, Murray 2005). Most studies raise the question of whether or not traditional point representation can effectively represent a demand region, such as neighborhoods, towns or census boundaries. Miller (1996) suggested that other spatial representations of objects, such as lines and areas (or polygons) in geographic information systems (GIS) should be considered in location modeling, and that there is a need for location modeling capable of dealing with more complex objects.

In this research, objects beyond points are explored for location coverage modeling. The optimal configuration of facilities represented as points are investigated in the plane to cover general demand objects. In the next section, point based representation problems are discussed. In addition, a few representation strategies beyond points are reviewed. This is followed by the introduction of a method for identifying a potential facility set for locating general objects. A warning siren coverage application is carried out to demonstrate the proposed method. Finally, a discussion and conclusions are provided.

2. Background

To represent the real world, the vector model in GIS utilizes three types of spatial objects: points, lines, and polygons. A point has no dimension and is the simplest representation of an object. Lines connect points and are often used to represent objects that are defined in one dimension, rivers or roads for instance. Polygons are used to represent areas of various sizes and shapes defined in two dimensions, such as census tracts, lakes and townships.

In location modeling, the region of interest is usually represented as a collection of points or as a regularly spaced series of points (Murray et al. 2006). Often, the demand region is discretized by dividing the area into small subareas, such as zones (or districts, census tracts, sectors, etc.), each represented by a demand point (Love et al. 1988). In part, it is due to the fact that it is easier to construct models based on finite points rather than on a geometrically complex region. Other reasons could be that point simplification makes data handling and solution procedures tractable in certain situations. As a result, the point representation of a region has long been used in location modeling.

Using a point with no dimension to represent an area of two-dimension can be viewed as an aggregation of infinite points to finite points (Francis and Lowe 1992). Aggregation creates a number of potential errors, including travel distance and coverage assessment errors, not to mention decreasing representational accuracy. As noted by Current and Schilling (1990), the error of aggregation of demand in covering models could be dramatic. For example, an area partially covered by a facility can be misclassified as completely covered or not covered at all based on the calculated distance of the aggregated point to the facility. These inaccuracies/errors are the result of unknowingly representing geographic space inappropriately through aggregation (Murray et al. 2006). Problems resulting from point representation of area demand in a covering problem are discussed in Murray and O'Kelly (2002).

Few studies can be found where more sophisticate shapes, such as areas, are used to represent regional demand in covering problems. Benveniste (1982) partitioned the demand region into rectangular subregions and studied the problem of coverage for the subregions. He concluded that the feasibility of solutions depends on how the region is partitioned and on the size of the subregions. Current and Schilling (1987) and Daskin et al. (1989) discussed the issue of spatial representation in covering models. However, in these works, area demand was aggregated into points.

3. Coverage of spatial objects

Covering models measure effectiveness through assessment of whether demand can receive service from located facilities (Church and ReVelle 1974). The key concept in these models is the acceptable proximity or coverage. Usually a maximum value, known as the service standard, is preset with respect to either distance or travel time, though the latter can be converted to a distance measure. Demand is said to be suitably served if it can be reached by any facility within this maximum coverage standard. For example, fire department response time should be less than 6 minutes to an accident or structure fire after a call for service has been received (Murray and Tong 2005).

In covering models, distance as a measure of relative spatial relation of the facility to demand is often used to evaluate coverage. The ℓ_p - distance measure is commonly used. The ℓ_p - distance in the plane is derived from the ℓ_p - norm, where $g(x) = \ell_p(q, r) = \sqrt[p]{|x_q - x_r|^p + |y_q - y_r|^p}$ for $1 \le p \le \infty$. (x_q, y_q) and (x_r, y_r) are points in the plane. The two most familiar and widely used distance measures, Euclidean distance and rectilinear or

Manhattan distance, are special cases of ℓ_p - distances with $p = 1$ and $p = 2$ respectively.

Suppose a facility F located at (x_f, y_f) that can provide service within a distance standard S, then any point demand (x_d, y_d) falling inside or on the boundary of the covering ball $\boldsymbol{B}$ = $\{d \mid \ell_p(d, f) \leq S\}$ can receive service from facility F. For Euclidean distance, the covering ball $\boldsymbol{B}$ is a circle centered on (x_f, y_f) with radius S, whereas for rectilinear distance, the covering ball is a diamond shape, as illustrated by Church (1984).

Assuming that a facility can be located anywhere in the plane and demand is point based, Church (1984) used a point set, known as circle intersect point set (CIPS), to serve as potential facility sites for solving covering location problems. He showed that this set contains at least one optimal solution under the Euclidean distance measure. Points in CIPS are the intersections of circles with radius S (or service standard S) centered on the point demands and hence are essentially intersections of boundaries of covering balls for these demands. The importance of this observation is that optimal solutions can be obtained by only examining points in CIPS instead of any point in the plane. This idea will now be extended to more general spatial objects such as lines and polygons.

Consider a line segment AB representing linear demand such as a street segment. End points A and B of segment AB are located at (x_A, y_A) and (x_B, y_B) respectively. If each point of A and B is within the covering ball $\boldsymbol{B}$, then any point on the line segment AB is also in the covering ball $\boldsymbol{B}$. That is, the ball $\boldsymbol{B}$ is a convex set and this has been proven based on the triangle inequality and homogeneity properties of norms (see Brimberg and Love 1995). This property indicates that if two end points of a line segment receive service from facility *F*, then the entire segment also receives service from *F*.

The coverage property of line segments can be extended for polygons or area objects. We observe that if each vertex of a polygon can receive coverage from facility *F*, then any point on the boundary or within the polygon can also receive coverage from facility *F*. This can be proven as follows. If all the vertices of polygon are covered by facility *F*, according to the line segment coverage property derived above any line segment connecting two vertices is also covered by *F*. Therefore the boundary comprised of series of line segments is also covered facility *F*. Now, consider an arbitrary point *M* inside the polygon. If a line is drawn passing through *M* in any direction, this line will intersect two or more line segments on the boundary. Since these intersection points are on the boundary and have been proven to be within the coverage of facility *F*, then point *M* on the line connecting these intersection points is also within the coverage of facility *F*. Thus, the entire polygon, boundary and interior, is covered when all vertices are covered.

Each polygon or line object has a covering boundary that can be derived and any facility located inside or on the covering boundary can provide complete coverage to the polygon. Figures 1 and 2 show the covering boundaries with rectilinear and Euclidean distance metrics for lines and polygons, respectively. In all the figures, if a facility is located in the overlap area, two demand objects can be covered. Otherwise, only one demand object can be covered.

Therefore, to achieve the maximal coverage, one strategy is to locate facilities in those overlap areas. The intersection points of the covering boundaries are in the overlap areas and can serve this objective. In fact, these intersection points derived by intersecting all the covering boundaries together with end points of lines or vertices of demand polygons, contain at least one optimal solution for covering problems, provided that the service standard *S* is greater than the diagonal of any line or polygon. We call this set polygon intersect point set (PIPS). The proof is a generalization of that provided by Church (1984).

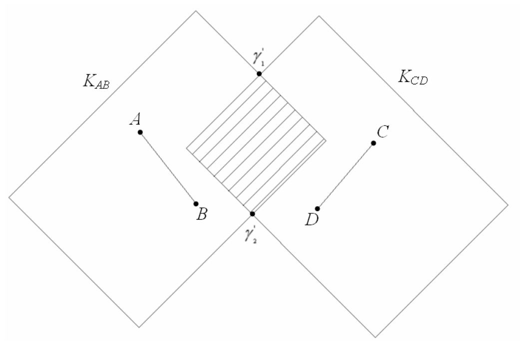

Figure 1a. Covering boundary for lines under rectilinear metric.

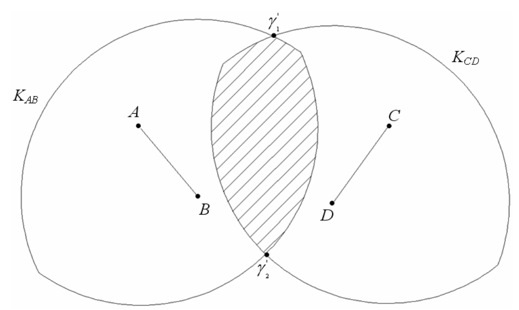

Figure 1b. Covering boundary for lines under Euclidean metric.

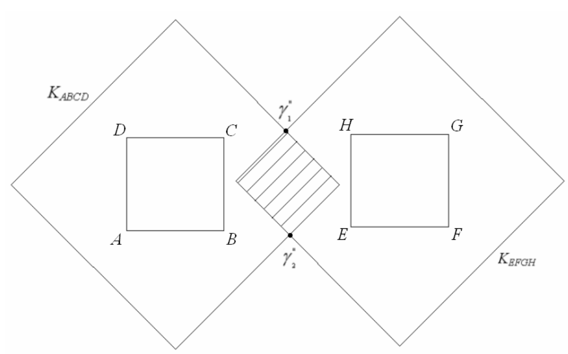

Figure 2a. Covering boundary for polygons under rectilinear metric.

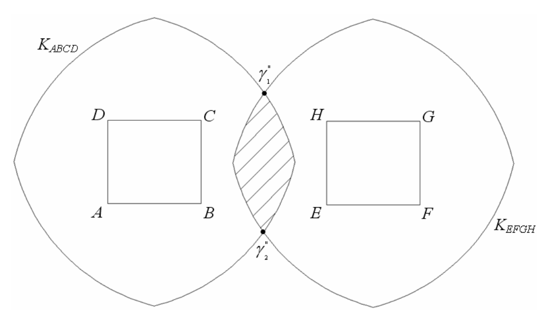

Figure 2b. Covering boundary for polygons under Euclidean metric.

These critical points (or PIPS) can be derived in a GIS environment. First, demand objects in terms of points, lines or polygons are identified. Then, a set of points including demand points, end points for lines or polygon vertices are extracted. Next, a coverage boundary for each object is derived. For a point, it is a buffer circle for the Euclidean distance metric and a diamond for a rectilinear distance metric. For a line or polygon, coverage boundaries for each vertex need to be generated first. The coverage boundary for the whole object is then the overlap of the vertex covering boundaries. The object covering boundary can be extracted using the overlay process in GIS. Finally, the critical points to serve as potential facility location can be identified by deriving the intersection points of the covering boundaries.

4. Application

The application of the PIPS for location analysis is demonstrated in this paper for warning siren siting in the city of Dublin, Ohio (shown in Figure 3). The city comprises parts of three counties, Franklin, Delaware and Union, with a total area of approximately 60 km^2 (23 square miles) and approximately 29,000 residents (1998). To site warning sirens, two factors are considered: one is that there is a nontrivial cost associated with purchasing and maintaining a warning siren as noted in Current and O'Kelly (1992) and Murray et al (2006); another is that individuals who fail to hear a warning during an emergency period are in danger. The service coverage in this area is provided by omni-directional sirens that have a maximum effective range of 976 meters.

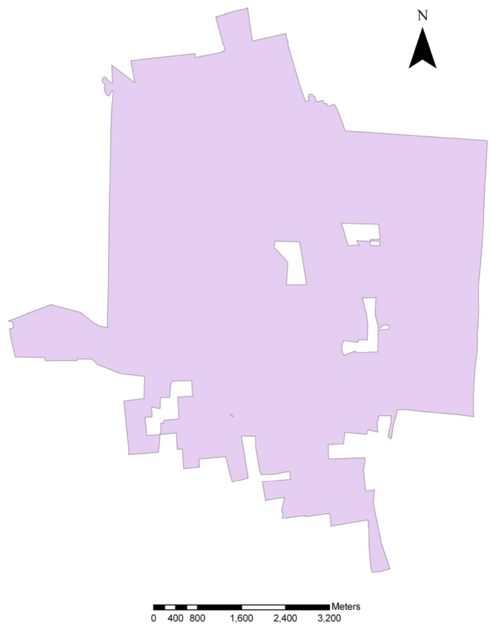

Figure 3. Study area of Dublin, Ohio

Our objective is to identify the minimum number of warning sirens needed to suitably cover this entire region. By delineating the city into 256 polygons, most 500mx500m in size, the problem becomes one to site sirens in continuous space to provide service to these polygons. Considering the omni-directional property of the sirens, the Euclidean distance measure is used to evaluate service coverage. By manipulating geometry operations (buffering, intersection, and overlay) in GIS, 4,232 critical potential facility location sites are identified in less than 10 minutes processing time on a Pentium Xeon 3.0GHz personal computer with 2.0GB of RAM. These derived points are then used in the Set Covering Problem – Spatial Object (SCP-SO) (Murray 2005) where β =0.5 & α =2. The result suggests twenty five sirens (shown in Figure 4) are needed to cover 99.94% of the total area. The model solution takes around 2 minutes using CPLEX.

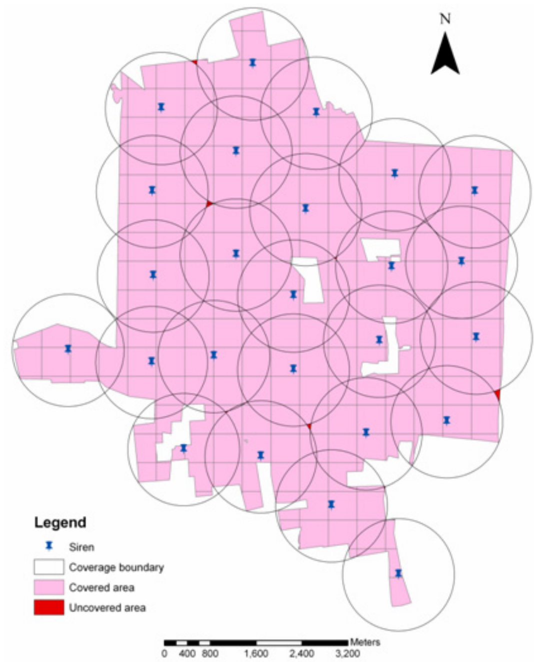

Figure 4. Identified siren configuration with PIPS.

A set of 4,584 regularly spaced points, 100m apart, in the region is also tested as potential facility locations in the model for comparison. Results are shown in Table 1. Twenty six sirens are needed to cover 99.91% of the whole region in this case. The time required to obtain the optimal solution is around 100 hours. That is, in this case one more siren is needed without improving coverage and more time is needed to obtain an optimal solution.

Potential facilities	No. of potential facilities	Solution	Actual coverage (%)	Branches	Iterations	Solution time (s)
PIPS	4,232	25	99.94	281	18,256	130.53
s100	4,586	26	99.91	5,109	1,052.69	360,192

*s100 is a set of regularly 100m-spaced points in the region

Table 1. PIPS and regular points as potential facility locations.

4,232 points generated randomly in the region are also tested for comparison. The random point set often takes much more time to solve. 50 different samples are evaluated. Compared to the PIPS results as shown in Table 2, more sirens are needed and more computation time is required for all the random points.

Case	Solution	Actual coverage (%)	Branches	Interactions	Solution time (s)
Best	26	100	19,835	1,989,406	5,795.52
Worst	27	99.73	621,100	58,554,541	399,723.58
Average	26.4	99.88	116,223	10,947,583	116,223.20

Table 2. Random points as potential facility locations.

Clearly, the PIPS are efficient and valid for use in this coverage modeling context. Compared with other alternative configurations in this application, PIPS identifies the fewest facilities needed with the least computation time. In addition, the identified facilities by PIPS provide almost complete coverage to the entire region.

5. Discussion and Conclusions

Through the use of GIS, this study extends traditional point-based space representation to a more complex and realistic form in covering location problems. A finite potential facility set - PIPS, which consists of at least one optimal solution for the covering problem for general demand, has been identified. Application results demonstrated that complex geometric shapes can be considered in covering models, such as site/service selection, in a way that remain computationally feasible.

Acknowledgements

This material is based upon work supported by the National Science Foundation under Grant No. 0518967. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

References

Benveniste R. 1982. A note on the set covering problem. *The Journal of the Operational Research Society* 33(3): 261-265

Birkin M., G. Clarke, M. Clarke and A. Wilson. 1996. *Intelligent GIS: Location decisions and strategic planning*. Cambridge: GeoInformation International.

Brimberg J. and R.F. Love. 1995. Estimating distances. In Drezner Z. (eds), *Facility location: a survey of applications and methods*.

Church R.L. 1984. The planar maximal covering location problem. *Journal of Regional Science* 24 (2): 185-200.

Church R.L. 1999. Location modeling and GIS. In P. Longley, M. Goodchild, D. Maguire, & D. Rhind (Eds.), *Geographical information systems* (second ed.)(pp, 293-303). New York: John Wiley.

Church R.L. and C. ReVelle. 1974. The maximal covering location problem. *Papers of the Regional Science Association* 32: 101-118

Current J.R. and D.A. Schilling. 1987. Elimination of source A and B errors in the p-median location problems. *Geographical Analysis* 19: 95-110.

Current J.R. and D.A. Schilling. 1990. Analysis of errors due to demand data aggregation in the set covering and maximal covering location problems. *Geographical Analysis* 22: 116-126

Current J.R. and M.E. O'Kelly. 1992. Locating emergency warning sirens. *Decision Science* 14: 47-54

Daskin M.S., A.E. Haghani, M. Khanal and C. Malandraki. 1989. Aggregation effects in maximum covering models. *Annal of Operations Research* 18: 115-140

Francis R.L. and T.J. Lowe. 1992. On worst-case aggregation analysis for network location problems. *Annals of Operations Research* 40: 229-246

Gleason J.M. 1975. A set covering approach to bus stop location. *Omega* 3: 605-608

Love R.F., J.G. Morris and G.O. Wesolowsky. 1988. *Facility location: models and methods*. NorthHolland, New York

Miller H.J. 1996. GIS and geometric representation in facility location problems. *International Journal of Geographical Information Systems* 10(7): 791-816

Minciardi R., R. Sacile and F. Siccardi. 2003. Optimal planning of a weather radar network. *Journal Atmospheric and Oceanic Technology* 20: 1251-1263.

Murray A.T. 2003. Site placement uncertainty in location analysis. *Computer, Environment and Urban Systems* 27: 205-21

Murray A.T. 2005. Geography in coverage modeling: exploiting spatial structure to address complementary partial service of areas. *Annals of the Association of American Geographers* 95(4): 761-772

Murray A.T. and M.E. O'Kelly. 2002. Assessing representation error in point-based coverage modeling. *Journal of Geographical Systems* 4:171-191.

Murray A.T., M.E. O'Kelly and R.L. Church. 2006. Regional service coverage modeling. *Computers and Operations Research*, to appear (special issue).

Murray A.T. and D. Tong. 2005. Analyzing structures fires in Massachusetts: regional science and the media. Manuscript submitted for publication

Murray A.T. and D. Tong. 2006. Coverage optimization in continuous space facility siting. Manuscript submitted for publication

ReVelle C. 1989. Review, extension and prediction in emergency service siting models. *European Journal of Operational Research* 40:58-65.

White A.N. 1979. Accessibility and public facility location. *Economic Geography* 55(1): 18-35.