# Vaccination Strategies for Communicable Diseases in a Dynamic Network of Human Contacts

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**Abstract:** Various network models of human contacts have been used to understand the efficacy of vaccination strategies. Few of them have explicitly taken the dynamics of the network into consideration. This paper explores efficient vaccination strategies in a dynamic network of human contacts. An individual-based spatially explicit model is used to represent the dynamic network. Its dynamics is described by two types of variations over time: the variation in individuals' spatial locations and the variation in individuals' contact numbers. Three vaccination strategies are simulated under the networks with different dynamics, including the random vaccination, the contact-targeted vaccination, and the distance-targeted vaccination. The simulation results show that vaccination strategies targeted to the individuals with the largest contact numbers or with the longest travel distances yield a great improvement of disease control. The reason is that these strategies make the diseases difficult to spread not only by reducing the mean contact number of a network, but also by altering the dynamics of the network. This research suggests that the explicit consideration of network dynamics in the vaccination strategies provides an alternative way to control communicable diseases efficiently.

**Keywords:** Dynamic Network, Vaccination, Communicable Diseases, Individual-based Model, GIS

## 1 Introduction

Vaccination is widely used to control the spread of communicable diseases in a human population (Dutta 2008). Communicable diseases usually spread from person to person via human contacts. The contact pattern among individuals in a population, i.e. the contact structure, is critical to the design of vaccination strategies (Eames and Keeling 2003, Shirley and Rushton 2005). From the view of the graph theory, the contact structure of a population can be properly modeled by a network composed of nodes and links. The nodes represent individuals and the links represent contacts among individuals (Wasserman and Faust 1994). The diseases spread over the network via the links. Vaccination endows individuals with the immunity to the diseases and removes them from the network (Madar *et al*. 2004).

A variety of network models has been presented to understand the disease spread and evaluate the vaccination strategies (Rhodes and Anderson 1996, Andersson 1998, Tuckwell *et al.* 1998, Watts and Strogatz 1998, Albert *et al.* 2000). These models have revealed a number of factors that can substantially facilitate the disease spread and thus are crucial for vaccination strategies (Newman 2002, Keeling 2005). The spatial location of an individual is one of the crucial factors. Individuals near to the source of infection may have higher risk of being infected and are suggested to be vaccinated first (Rhodes and Anderson 1997, Longini *et al.* 2005). Another crucial factor is the number of contacts of an individual. Individuals with the largest number of contacts act as hubs to distribute the infection, and are suggested to be vaccinated first (Pastor-Satorras and Vespignani 2002, Zanette and Kuperman 2002).

In most of these studies, the spatial location and the contacts of an individual remain constant over time (Keeling and Eames 2005). In other words, the vaccination strategies are studied primarily based on static networks. It is our intuitive perception that the human contacts form and break over space and time (Golledge and Stimson 1998). This dynamics should not be ignored in the design of vaccination strategies (Keeling and Eames 2005). In addition, recent studies have pointed out that the network dynamics could independently leads to the persistence or decline of an epidemic (Bian and Liebner 2007). The lack of considering this may mislead our understandings of vaccination strategies (Dye and Gay 2003, Galvani 2004). Those vaccination strategies efficient in a static network might perform distinctly in a dynamic network. Few studies on vaccination strategies have explicitly taken the dynamics of the network into consideration.

The purpose of this paper is to explore efficient vaccination strategies in a dynamic network of human contacts. An individual-based spatially explicit model (Bian 2004) is used to represent the dynamic network. The dynamics of the network is described by the temporal variations in two crucial factors for vaccination strategies, i.e. the individuals' spatial locations and their contact numbers. A set of vaccination strategies are simulated using the networks with different dynamics and their efficacies are evaluated. The results could provide an evaluation of vaccination strategies that are close to the real situations. They could also help understand the relationship between the dynamics of network and the efficacy of vaccination strategies. The reminder of this paper begins with the representation of a dynamic network and the description of its dynamics. The section that follows describes the design of vaccination strategies for the dynamic network. The results of the vaccination strategies are discussed.

## 2 The dynamic network of human contacts

The network of human contacts is intrinsically dynamic. Individuals may contact with different groups of individuals at different time and locations because of their mobility (Hägerstrand 1970). This dynamic network of human contacts can be represented by an individual-based spatially explicit model presented in Bian (2004). The model is conceptualized as a two-tier and two-level network (Figure 1). The two tiers refer to the home space in nighttime and the workplace space in daytime, respectively. Individuals have explicit locations (with *x*, *y* coordinates) in their home space and workplace space. The two levels refer to the human contacts at the local and population level, respectively. Individuals commute between homes and workplaces on a daily basis, which is represented as individual lifeline (Lenntorp 1977, Miller 2005). These lifelines intersect at homes and workplaces, forming a set of local level networks at each tier. The individuals' travel between their homes and workplaces connects the two sets of local networks into a population-level network. Based upon such model, the dynamics of the network can be described by two types of temporal variations between daytime and nighttime. One is the temporal variation in the spatial locations of an individual and the other is the temporal variation in the contact numbers of an individual. For short, the former is termed as the variation in locations, and the latter as the variation in contact numbers.

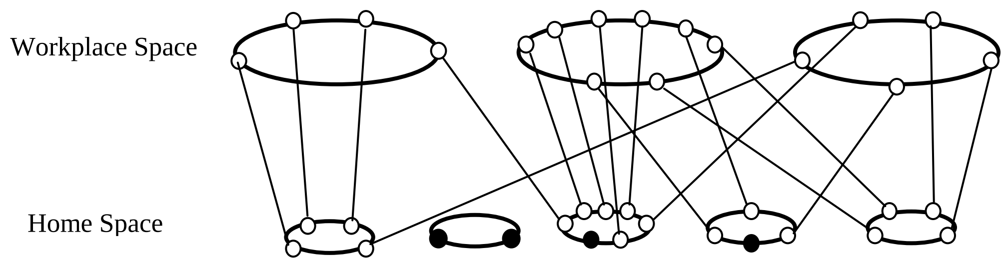



**Figure 1.** The conceptualized two-tier and two-level network (Bian 2004). Ovals at each tier represent homes, workplaces and service places, respectively. Small circles on an oval represent individuals who interact with each other within a location. A straight line among tiers is a lifeline that links an identical individual at different locations.

In the context of the graph theory, the structure of a network can be measured by a set of parameters, such as the mean number of links per node, the minimum number of links between any pair of nodes, the degree of connection between a given number of nodes, and so on (Watts and Strogatz 1998, Albert *et al.* 2000). These parameters can be extended to measure the contact structure among individuals (Bian 2004, Eubank *et al.* 2004). In this paper, two network parameters introduced by Bian and Liebner (2007) are used to measure the two types of aforementioned variations, respectively. One is the covariance of an individual's workplace location and home location, for *x* and *y* coordinates, respectively. This parameter was originally used to measure the degree of interconnection of the network. Here, it is used to measure the variation in locations of a network. A lower covariance value indicates that an individual's home location is more distinct from his/her workplace location. In other words, the individual's home location is farther away from the workplace location. The network therefore has a stronger variation in locations. The other parameter is the ratio between an individual's workplace contacts to home contacts. This ratio was originally used to contrast the local networks in the workplace space with those in the home space. Here, it is used to measure the variation in contact numbers of a network. A larger contact ratio indicates that an individual has more contacts at workplaces in the daytime than those at homes in the nighttime. The network therefore has a stronger variation in contact numbers.

A range of networks with different dynamics can be defined by altering these two parameters, and can be related to the contact structure of real communities (Figure 2). Figure 2(a) and 2(b) exhibit a dynamic network with weak variation in contact numbers (Contact ratio =1:1) but strong variation in locations (Covariance =0). Individuals have approximately the same number of home contacts in nighttime and workplace contacts in daytime. Most individuals work far from their homes. This type of network may represent a community in a small town or city, where the workplace size is relatively small but people often use private vehicles or take public transportations to work. Figure 2(a) and 2(c) show the network of weak variations in both contact numbers (Contact ratio =1:1) and locations (Covariance =0.99). The workplace size is similar to the home size, and individuals' workplaces are near to their homes. This type of network may correspond to a village in the rural area. Farmers live and work in their farms.

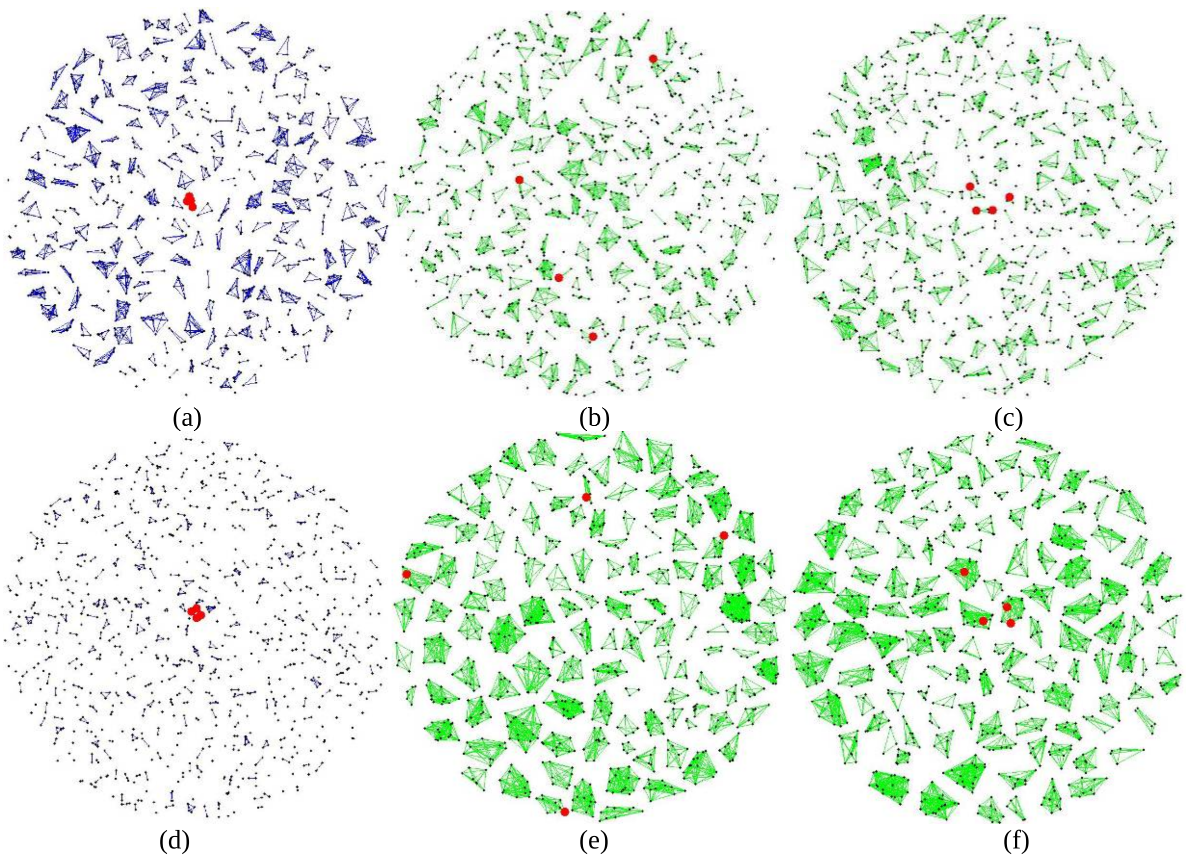


**Figure 2.** An illustration of the networks with different dynamics. Dots represent individuals and lines represent their contacts. Red dots represent individuals from a same family. In the first line, (a) is the home space in nighttime given contact ratio = 1:1; (b) and (c) are the workplace spaces in daytime, under the same contact ratio as (a), but different covariance with (a) at 0 and 0.99, respectively. In the second line, (d) is the home space in nighttime given contact ratio= 7:1; (e) and (f) are the workplace spaces in daytime, given the same contact ratio as (d) but different covariance with (d) at 0 and 0.99, respectively.

Figure 2(d) and 2(e) illustrate another type of network with strong variations in both contact numbers (Contact ratio =7:1) and locations (Covariance =0). Note that the workplace size is apparently larger than the home size. In addition, individuals' workplaces are far from their homes. This type of network represents communities such as those in the suburban of a metropolitan, where people may work in large workplaces in certain parts of the urban area but live in the suburban. Figure 2(d) and 2(f) show the network with strong variation in contact numbers (Contact ratio =7:1) but weak variation in locations (Covariance =0.99). The workplace size is still apparently large but people work close to their homes. This type of network may represent a community near to the downtown area of a metropolitan, where people work in large workplaces and their homes are close to the workplaces.

Networks with different dynamics also produce distinct patterns of disease spread. A higher contact ratio (stronger variation in contact numbers) leads to a lower epidemic peak (the maximum number of daily new infected individuals) and later peak time (the time duration to reach the peak)(Bian and Liebner 2007). The reason is that a higher contact ratio means a larger size for workplaces and a smaller size for families. Due to the small family size at one end, the disease spread throughout the population is primarily dependent on the workplace contacts. This situation makes the entire network inefficient and difficult for diseases to spread. On the other hand, a lower covariance (stronger variation in locations) produces a higher epidemic peak and earlier peak time (Bian and Liebner 2007). A lower covariance indicates that individuals' home locations differ from their workplace locations. It implies that individuals may contact with a greater number of different individuals between daytime and nighttime—lower degree of network interconnection. Therefore, the disease could spread more easily and rapidly in the network (Watts and Strogatz 1998, Keeling 1999). The effects of network dynamics on the disease spread may provide insight into the design of vaccination strategies for dynamic networks, which are described in the following sections.

## 3 Vaccination Strategies in the dynamic networks

In this section, two targeted vaccination strategies and one random vaccination strategy are considered. The two targeted vaccination strategies involve the contact-targeted vaccination and the distance-targeted vaccination. The former aims to vaccinate a fraction of population that has the largest number of contacts before an infection starts. The latter aims to vaccinate a fraction of population that has the longest travel distance between homes and workplaces before an infection starts. For comparison purpose, the random vaccination is also considered, which aims to randomly vaccinate a fraction of population before an infection starts.

To discuss its efficacy, each of the three vaccination strategies is simulated in a dynamic network with a total population of 2000. The population is simply assumed to be closed (there are no births, deaths, or migrations during the epidemic). The mean contact number per individual (daytime plus nighttime) is set to 10. For the vaccination process, the vaccination fraction is given a range of values, from 0% to 40% of the total population. This can help to investigate what fraction of population must be vaccinated to suppress the epidemic. The 0% vaccination fraction represents the no-response scenario, which is used as a baseline for comparison. On the other hand, the dynamics of the network is specified by giving alternate values to either the contact ratio or the covariance of the network, while holding the other constant. This can help to illustrate the independent effect of either parameter. The default value is 3:1 for the contact ratio, and 0.5 for the covariance. Each vaccination strategy is evaluated under the combined effects of the vaccination process and the network dynamics, parameterized pairwisely by a vaccination fraction associated with either a contact ratio or a covariance.

An index called epidemic slope is presented to evaluate the efficacy of each vaccination strategy. The epidemic slope is derived from the epidemic curve, which depicts the number of daily infected individuals during the epidemic (Figure 3). The index is defined as the size of an epidemic peak divided by the peak time in days. Graphically, it approximates the slope of an epidemic curve. A larger value of the epidemic slope indicates a higher epidemic peak and earlier peak time, which further implies faster transmission of the diseases. The epidemic slope produced by each vaccination strategy will be the average results of 100 realizations via Monte Carlo method under the pair of parameters. The network structure will be constructed anew for every 10 realizations

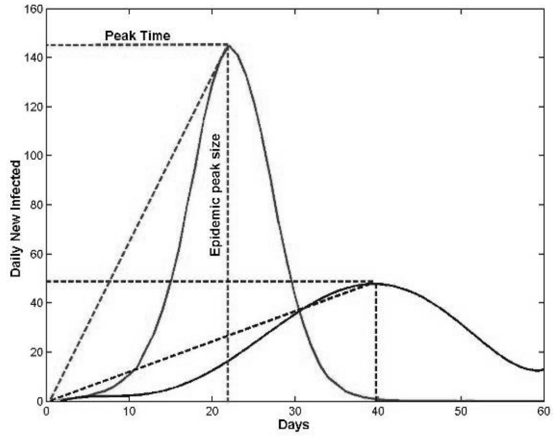



**Figure 3** The epidemic curves and slopes are used to evaluate vaccination strategies. Larger value of the epidemic slope indicates higher epidemic peak and earlier peak time, which further implies faster transmission of the diseases.

In the simulation, each individual is associated with an infection status, which can be either susceptible, or infected, or recovered (Kermack and McKendrick 1927). Influenza is chosen for the simulation because it is common and readily communicable between individuals. The infection rate is set to 0.1, which is the probability of a susceptible individual being infected once in contact with an infected individual. The influenza virus is assumed to be latent for one day in an infected individual, and then it can be transmitted to other susceptible during the following four days. After that, the infected individual gets recovered and is immune to the infection during the epidemic. The infection process is simulated by a bi-daily time step between daytime and nighttime over 60 days. A vaccination strategy is applied before an infection starts, vaccinating a fraction of population and removing them from the network. Then, one infected individual is introduced to initialize the disease spread in the network.

## 4 Simulation results

The results of this paper are presented in two sub-sections: 1) the performance of contact-targeted vaccination against the variation in contact numbers, and 2) the performance of distance-targeted vaccination against the variation in locations. To further explain the results, the structural changes in the network by the vaccinations are also investigated in each sub-section.

### Contact-targeted vaccination against the variation in contact numbers

Figure 4 demonstrates how the contact-targeted vaccination and the random vaccination perform upon the networks with different variations in contact numbers, from weak (contact ratio =1:1) to strong (contact ratio =9:1). Given a fixed contact ratio, a higher vaccination fraction leads to a lower epidemic slope for both vaccination strategies. The more individuals get vaccinated in advance, the harder the disease can spread in the population. A vaccination fraction of 40% is found to be sufficient for both vaccination strategies to suppress the epidemic in a network with any contact ratio. On the other hand, given a constant vaccination fraction, the epidemic slope is decreasing as the contact ratio rises. This trend is consistent with our expectation that higher contact ratio leads to a lower epidemic peak as well as later peak time. Comparing the two vaccinations, it is easy to find that the contact-targeted vaccination always produces lower epidemic slopes than the random vaccination dose if they are applied under the same vaccination fraction. Their difference in epidemic slope is apparent in the network of low contact ratio (e.g. 1:1 and 3:1), but it is converging as the contact ratio increases to 9:1.

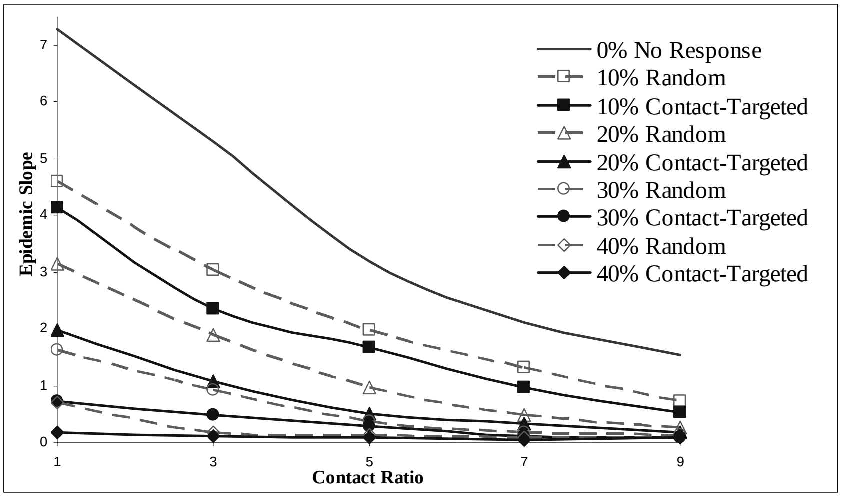
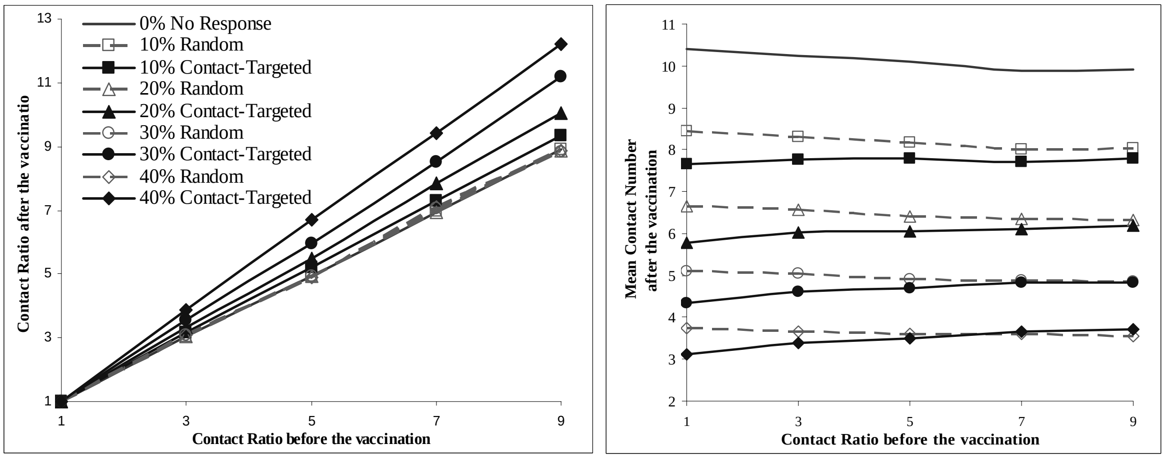



**Figure 4** The epidemic slopes for vaccination strategies against contact ratios and vaccination fractions. The confidence intervals are smaller than the symbol size and are omitted for clarity.

To further explain their difference, the structural changes of the network by both vaccinations are investigated. Figure 5 demonstrates the changes in two parameters of the network before and after the vaccination, the contact ratio (Figure 5a) and the mean contact number (Figure 5b). The no-response scenario shows no changes in the two parameters before and after the vaccination, because no individuals have been removed from the networks. Similarly, no changes are found in the contact ratios before and after the random vaccination (Figure 5a). This is universal for any vaccination fraction and any contact ratio given before the vaccination. However, the contact ratios are raised after the contact-targeted vaccination. The raise is apparent for a higher vaccination fraction or a higher contact ratio given before the vaccination. Both vaccination strategies can reduce the mean contact number after they are applied (Figure 5b). The higher the vaccination fraction, the lower the mean contact number would be after the vaccination. The contact-targeted vaccination can produce lower mean contact number than the random vaccination dose, but the difference is converging as the contact ratio rises.




(a) Contact ratio (b) Mean contact number

**Figure 5** Network structural parameters before and after the vaccination. The horizontal axis represents the contact ratio of a network before the vaccination. The vertical axis represents two network parameters after the vaccination.

The results imply that the random vaccination has no effects on the variation in contact numbers of a network. It controls the disease spread only by reducing the mean contact number of the network. The contact-targeted vaccination not only reduces the mean contact number of the network, but also intensifies the variation in contact numbers. This explains why the contact-targeted vaccination outperforms the random vaccination in a dynamic network. The converging trend in the mean contact number produces by both vaccinations, as shown in Figure 5(b), explains why their difference in epidemic slope converges as the contact ratio increases. In the networks with high contact ratios (e.g. 7:1 or 9:1), both vaccination strategies reduce the mean contact number to a similar level. But the contact-targeted vaccination could additionally intensify the variation in contact numbers, and thus still produces lower epidemic slopes.

**Distance-targeted vaccination against the variation in locations**

Figure 6 demonstrates the performance of both distance-targeted vaccination and random vaccination upon the networks with different variations in locations, from strong (covariance =0) to weak (covariance =0.99). Vaccinating 40% of the population would be sufficient for both vaccination strategies to suppress the epidemic in a network with any covariance value. For a given vaccination fraction, the epidemic slope is decreasing as the covariance rises. This trend is consistent with our expectation that a higher covariance leads to a lower epidemic peak as well as later peak time. Interestingly, there exists a threshold of the covariance, above which the epidemic slope drops drastically. This threshold is approximately 0.8 for the no-response scenario and the random vaccination. But it is lower for the distance-targeted vaccination, approximately at 0.5. Thus, the distance-targeted vaccination results in earlier and dramatic drop in the epidemic slope as the covariance increases. It produces lower epidemic slopes than the random vaccination dose after the covariance exceeds its threshold.

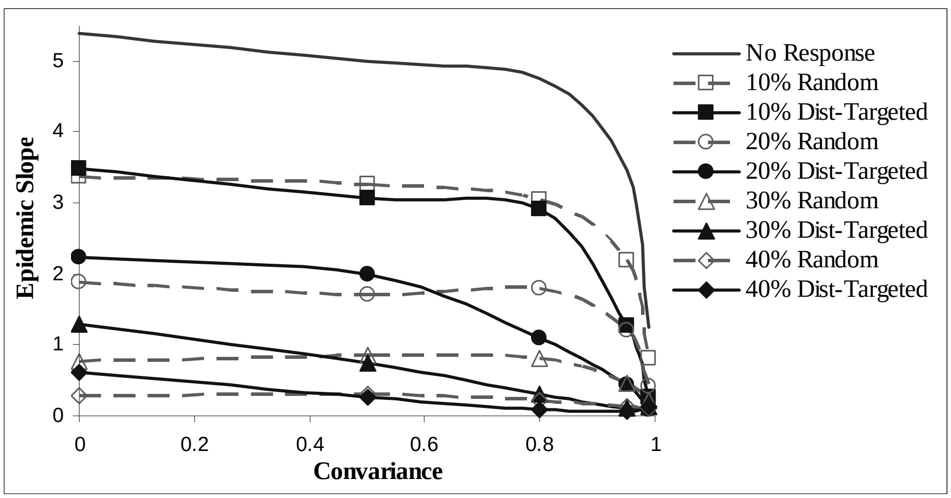
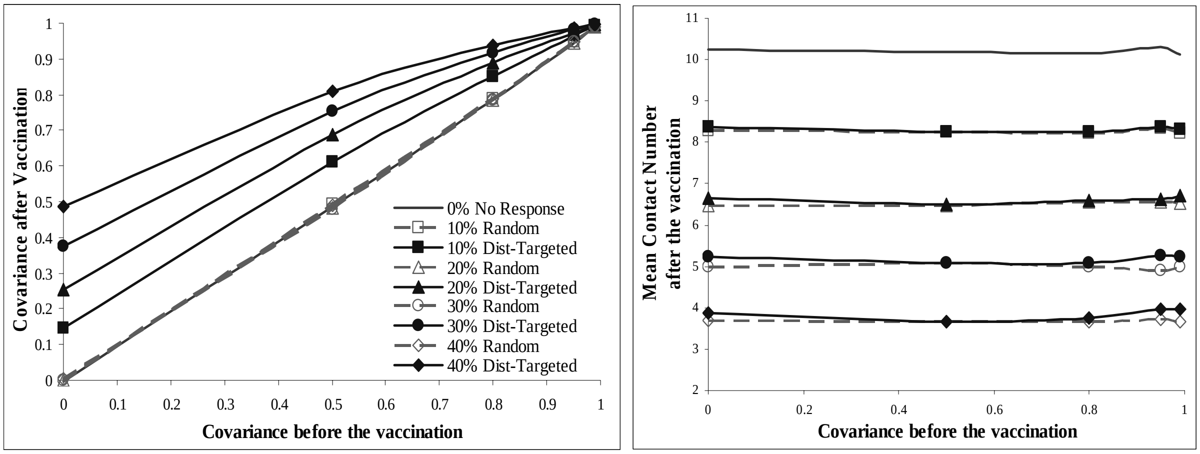



**Figure 6** The epidemic slopes for vaccination strategies against the network covariance and vaccination fractions. The confidence intervals are smaller than the symbol size and are omitted for clarity.

To further explain this earlier drop, the structural changes of networks by both vaccinations are also investigated. Figure 7 demonstrates the changes in two parameters of the network before and after the vaccination, the covariance (Figure 7a) and the mean contact number (Figure 7b). Again, the no-response scenario indicates no changes in network parameters before and after the vaccination. As shown in Figure 7a, no changes happen in the covariance before and after the random vaccination. This is universal for any vaccination fraction and any values of covariance given before the vaccination. In contrast, the covariance is raised after the distance-targeted vaccination. This is especially apparent for a higher vaccination fraction or a lower covariance value given before the vaccination. Both vaccination strategies can reduce the mean contact number after they are applied (Figure 7b). The random vaccination produces the mean contact number a little lower than the distance-targeted vaccination dose.




(a) Covariance (b) Mean contact number

**Figure 7** Network structural parameters before and after vaccination. The horizontal axis represents the covariance of a network before the vaccination. The vertical axis represents two network parameters after the vaccination.

The results imply that the random vaccination has no effects on the variation in locations of the network. It controls the disease spread purely by reducing the mean contact number of the network. The distance-targeted vaccination not only reduces the mean contact number of the network, but also weakens the variation in locations. Given the covariance of the network is above 0.5 before the vaccination, the distance-targeted vaccination can raise the covariance to a higher level (close to 0.8). This dramatically weakens the variation in locations of the network, and individuals may contact with same groups of individuals between daytime and nighttime. This situation makes the entire network inefficient for disease transmission (Keeling 1999). Therefore, the distance-targeted vaccination leads to dramatic drop in the epidemic slope earlier than the random vaccination. This explains why the distance-targeted vaccination is quite efficient as the covariance exceeds 0.5, even though it produces a little higher mean contact number.

## 5 Conclusions and Discussions

This paper focuses on exploring efficient vaccination strategies in the dynamic networks, rather than those in the static networks that have been well-studied. Three vaccination strategies are simulated upon a modeled dynamic network of human contacts. The contact-targeted vaccination and the distance-targeted vaccination are designed based on the heterogeneity among individuals in terms of their contact numbers or travel distances, respectively. Their efficacies in disease control are compared to that of the random vaccination, which treats individuals homogenously.

The simulation results show that the random vaccination can only reduce the mean contact number of the network, but has no effects on the dynamics of the network. Both targeted vaccinations can also reduce the mean contact number of the network. Furthermore, the contact-targeted vaccination is able to intensify the variation in contact numbers of a network, while the distance-targeted vaccination could weaken the variation in locations of a network. Thus, they could achieve greater efficacy to control diseases than the random vaccination. Our results suggest that vaccination strategies that are able to alter the network dynamics could help improve the efficacy of disease control. Besides reducing the mean contact number of a network, the explicit consideration of network dynamics provides us an alternative way to design vaccination strategies.

From a practical perspective, an optimal vaccination strategy needs to consider the trade-offs between its efficacy and costs (Holme 2004, Gomez-Gardenes *et al.* 2006). Both targeted vaccination strategies show greater efficacy in disease control, but they also require knowledge about the contact network and a well-developed logistic system (Cohen *et al.* 2003). This demands the constant updates of information about the local demographics, workplaces, and travel patterns in order to understand the network dynamics. A well-developed logistic system is also required to identify the targeted population and allocate health resources. Generally, both targeted vaccinations would be the optimal choice when the health resources are limited, which usually are. Specifically, the contact-targeted vaccination is preferable for the networks with moderate or weak variation in contact numbers, such as the small cities and rural areas. The distance-targeted vaccination would be preferable for the networks with moderate or weak variation in locations, such as the communities near to the downtown areas of a metropolitan. Under these situations, they would achieve greater efficacy of disease control with the minimum costs.

On the other hand, the random vaccination could weigh over the two targeted vaccinations under two situations suggested by the results: 1) the health resources are sufficient, and a high vaccination fraction (greater than the threshold 40%) can be achieved; 2) the network has extremely strong variations in both locations and contact numbers, such as the suburban areas. Under these two situations, the random vaccination could achieve approximately the same (even greater) efficacy as the two targeted vaccinations. More importantly, it is much easier to apply and may not need the knowledge about the contact network.

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