Simultaneous Changepoint Detection in Temperature Data

Steven Moen and Yuhan Liu
The University of Chicago, Department of Statistics

Abstract

Motivation for this work came from monthly temperature readings taken across the United States, which seem to be more tightly linked to each other in the mid-2000s than they were in prior decades. To analyze this trend more carefully, we have created a test statistic that allows for inference about whether distinct time series have a simultaneous change point, which for our purposes means a change in the first derivative in two different series at the same point in time.

This statistic can be used to measure the likelihood that two temperature time series are changing at the same time or whether the change is simply caused by random chance.

The statistic does not have a closed-form analytical density, so Monte Carlo simulation is required for hypothesis testing. Developing a proper simulation study to precisely measure the noise in the underlying data is beyond the scope of this poster, but the statistic can be used to measure change in many areas of earth science in addition to temperature.

Introduction and Motivation

Using average temperature data gathered from 15 locations across the U.S. from the U.S. Historical Climatic Network [1] and fitting local quadratic models shows some evidence of an increase in the rate of warming in the mid-2000s.

Test Statistic Derivation

Consider two sets of observations indexed by time: 

\[ (X_t, Y_t), \ldots, (X_T, Y_T) \]

To measure whether they have a similar changepoint in their first derivative, the first step is to split each time series at a hypothesized changepoint and fit two separate quadratic regression models to each series:

\[ X_t = a_0 t^2 + a_1 t + a_0 \quad Y_t = b_0 t^2 + b_1 t + b_0 \]

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From these models, their analytical first derivatives can be calculated as follows:

\[ X'(t, 1) = 2a_0 t + a_1 \quad Y'(t, 1) = 2b_0 t + b_1 \]

\[ X'(t, 2) = 2a_0 t + a_1 \quad Y'(t, 2) = 2b_0 t + b_1 \]

After calculating these above first derivatives, the average of two different slopes connecting three consecutive points would be calculated as follows:

\[ \bar{X}_i = \frac{X_{i+1} + X_{i-1} - 2X_i}{2} \quad \bar{Y}_i = \frac{Y_{i+1} + Y_{i-1} - 2Y_i}{2} \]

After finding these two sets of values, they would be compared in using the residual sum of squares (RSS) statistic for every possible change point. The calculation necessary to find the RSS is given below:

\[ RSS_X = \sum_{t=1}^{T} (\bar{X}_i - X'(t, 1))^2 + \frac{T}{T-1} (\bar{X}_i - X'(t, 2))^2 \]

\[ RSS_Y = \sum_{t=1}^{T} (\bar{Y}_i - Y'(t, 1))^2 + \frac{T}{T-1} (\bar{Y}_i - Y'(t, 2))^2 \]

Comments on the Test Statistic

- The test statistic \( K \) does not have a closed-form analytical density without the highly unrealistic assumption of independence between the terms used to calculate \( K \).
- \( K \geq 0 \) because the model assuming a changepoint at the same time is inherently simpler than a model assuming different changepoints.
- Despite an extensive literature search, it looks like Monte Carlo testing is required for statistical inference, which is beyond the scope of this poster.

Future Work and Acknowledgements

Detecting changepoints in real data is difficult, and Monte Carlo simulation is necessary to construct a null distribution for hypothesis testing. To conduct proper Monte Carlo analysis, a realistic theoretical model is required, so combining this test statistic with existing, sound theoretical models will allow for powerful insights into changepoints. The test statistic could be generalized to conduct more than just pairwise comparisons.

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References

